

Recovery of Relativistic Celestial Trajectories as an Effective Field Limit of Internal Momentum Flux

A Flat-Space Kinematic Account of the Classical Tests

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Abstract

General Relativity attributes gravitational trajectory deflection to the pseudo-Riemannian curvature of a four-dimensional spacetime manifold, and so contains two kinds of entity: matter and geometry. In this paper, I show that the classical single-centre tests of relativistic gravitation are recovered on an uncurved flat Minkowski background $(\mathbb{R}^{1,3}, \eta_{\mu\nu})$ with only one kind of entity, momentum flux, from three short axioms, two stated rules and one calibration, G (Table 12). The phenomena recovered include the anomalous $43.0''$ /century perihelion precession of Mercury, the $1.75''$ achromatic deflection of starlight, the logarithmic Shapiro time delay, geodetic precession and frame dragging of orbiting gyroscopes, and the uniform cadence of operational clocks on the terrestrial geoid.

The axioms are: (I) a flat Minkowski substrate with invariant speed of light c ; (II) matter as confined speed- c momentum flux, with a single speed budget shared by all circulations; and (III) additivity and local conservation of momentum flux, with all flux, confined or free, propagating at the local speed set by the unified debt, only debt relative to the ambient level accessible, and the unified distribution read out once. Additivity fixes the Green's function of the framework: each confined source projects a retarded, null stream whose arc curvature $1/L$ is the geometric sagitta residual, and the stream deposits its debt along its arrival direction. Integrating this Green's function directly over an extended body derives the exact inverse-square drift strictly as a property of harmonic volume averaging for the 1st moment ($\mathbf{N}$), proving that Newtonian point-mass reduction is an invalid cancellation that fails for the arrival tensor T_{ij} . The arrival tensor preserves internal geometric memory of the body's interior ($\langle r'^2 \rangle$), breaking Birkhoff's theorem and General Relativity's metric blindness by generating transverse hoop stress near an extended body, while total momentum flux conservation ($\partial_\mu \Pi^{\mu\nu} = 0$) and spherical parity forbid time-dependent monopole radiation by construction.

The sagitta operates as a single continuous geometric mechanism: the microscopic singular sagitta of each circulation generates the retarded $1/L$ Green's function, while the relational sagitta of constituent pairs expresses the pairwise angular interference of arriving flux across $N(N-1)$ directed channels, carrying the body-scale geometry and enforcing bilateral momentum conservation. The exterior dynamics are governed by a single continuous equation of geometry and energy, where the $\sqrt{2}R$ threshold marks the transition from purely additive relational momentum (acute aperture, $\Theta < 90^\circ$, $\cos \theta_{bc} > 0$, Region 1) to destructive limb-curl cancellation ($\Theta > 90^\circ$, $\cos \theta_{bc} < 0$, Region 2), which softens the dynamic terms on bodies with bulk momentum entering Region 2 and deposits the cancelled vector momentum as internal stress and constituent dilation. At first order the sagitta map and the rod-clock readout are joined by an exact dictionary, generated by the distribution itself, for any distribution of sources. The dynamics consist of two non-overlapping operations, linear causal projection and local adiabatic relaxation, followed by motion; relaxation follows from additivity and universal confinement, giving an exponential response and $\beta = 1$. A first-order consistency check selects $\gamma = 1$; the kernel shows no aberration for uniformly moving sources, no preferred frame at linear order ($\alpha_1 = \alpha_2 = 0$), frame dragging, and tensor radiation, and light and gravitational ripples share one local light cone, as GW170817 requires.

Across all currently verified empirical regimes, the framework is empirically equivalent to General Relativity; physical departures are strictly demarcated to near-limb Region 2 dynamic softening and asymptotic strong-field compression (matter decelerates in cadence and compresses without freezing, yielding no event horizon or singularity). Within the verified empirical baseline, its claims are of explanatory economy.

Contents

1	Introduction	5
1.1	Orbital Causality and the Geometric Problem	5
1.2	From Compton Phase to Celestial Trajectories	6
1.3	Architecture: Two Branches and One Operator	6
1.4	Epistemic Scope and Domain Demarcation	7
1.5	Relation to Prior Work	8
2	Axiomatic Foundations	9
2.1	Stated Rules and Calibrations	10
3	Reader's Map: From Expectations to This Framework	11
4	Valid and Invalid Cancellation	12
5	Branch A: The Green's Function	13
5.1	Source-Centred Arcs and the Singular Sagitta	13
5.2	The Kernel and Its Three Parts	13
5.3	From the Sagitta to the Inverse-Square Drift	14
5.4	Tangent-Space Descriptions and the Order of Summation	15
5.5	Moving Sources: No Aberration	15
5.6	Synchronization of an Extended Source	15
5.7	The Radiation Ledger	15
6	The Unified Sagitta Mechanism: Microscopic Generator and Relational Pairwise Ledger	16
6.1	Microscopic Generator: The Singular Sagitta	16
6.2	The Directed Relational Ledger and Newton's Third Law	16
6.3	The Arriving Angular Identity at a Target	17
6.4	Closure at Directed Pairs: The Triad as Multi-Body Demonstration	18
7	Direct Integration over an Extended Body	18
7.1	The Scalar Debt: The Monopole as a Result	18
7.2	The Arrival Tensor: Exact, with One Terminating Term	18
7.3	Momentum Flux Is a Conserved Tensor	19
7.4	Which Moment the Shell Theorem Governs	19
7.5	The Relational Departure from Birkhoff's Theorem and Metric Blindness	20
8	The Bridge between Branch A and Branch B	20
9	Three Regions	21

10 Branch B: Composition and Relaxation	23
10.1 Three Operations with Non-Overlap	23
10.2 Relaxation Is Additivity	23
10.3 The Universal Confinement Response	24
10.4 Relaxed Emission and the Active Mass	24
10.5 The Supplied Initial Condition and the Fold-Back	25
10.6 Rearrangement and Path Independence	25
10.7 Back-Reaction as Coarse-Graining	25
10.8 The Summed Green's Function and the Momentum-Flux Balance Law	25
10.9 The First-Order Single-Source Geometry (Kerr-Schild Form)	26
10.10 Synchronization and the Single-Centre Readout	27
10.11 Momentum Conservation	27
11 The Matter Interface	28
11.1 Six Class Properties	28
11.2 Persistence Gives Weight Equal to Inertia	28
11.3 A Worked Transit Loop	29
11.4 Particles Carry the Flux; Atoms Organise It	29
11.5 The Separatrix and the Control Surface	29
11.6 Hydrostatic Balance	29
12 Rod-Clock Kinematics, the Interior, and the Light Cone	29
12.1 Operational Definitions	29
12.2 Debt, Local Streaming, and Instrument Verification	30
12.3 The Areal Compensation and the Interior	30
12.4 Loop Orientation Invariance	31
12.5 The Moving Loop and the Speed Budget	31
12.6 Wave Mechanics: The Eikonal Light Cone	32
13 The Classical Tests	32
13.1 Perihelion Precession of Mercury	32
13.1.1 The velocity-dependent scalar-potential barrier	32
13.1.2 Relaxation of an orbiting assembly	32
13.1.3 Chart dependence of the precession partition	33
13.1.4 Perturbative solution of the Binet equation	33
13.2 Achromatic Light Deflection	34
13.3 Radar Shapiro Delay	34
13.4 Strong Fields: Photon Sphere, Shadow, and Horizon (Conjectures)	34
13.5 Terrestrial Manifestation: The Geoid	35
13.6 Geodetic Precession and Frame Dragging	35
14 PPN Parameters, Binding, and Multi-Source Readout	35
14.1 PPN Parameters	35
14.2 Free Fall, Active Mass, and the Nordtvedt Parameter	35
14.3 What Superposes: Relational Readout versus Metric Superposition	36
14.4 Instrument Readout in Multi-Source Fields	36
14.4.1 Relational Free Fall	36
15 Parsimony and Explanatory Economy	36

16 Empirical Demarcation and Domain Boundaries	37
16.1 Empirical Equivalence Across All Current Tests	37
16.2 The Two Domains of Physical Departure	39
16.3 Computational and Astrophysical Programme	39
17 Conclusion	40
A Momentum Flux Mathematics	40
A.1 Shell Averages and the Monopole	41
A.2 The Arrival Tensor	41
A.3 The Tensor Conservation Law	41
A.4 The Absolute-Net Identity and Closure at Directed Pairs	42
A.5 Onset of Opposed Pairs	42
A.6 The Bridge Identity and the Synchronization Gradient	42
A.7 The Retarded Distance of a Uniformly Moving Source	42
A.8 The Virial Identity	42
A.9 The Relative-Debt Functional Equation	42
A.10 The Eikonal Light Cone	43
A.11 Parity and Conservation for a Spherically Symmetric Source	43
A.12 Numerical Checks	43
B Summary Tables	44

1 Introduction

1.1 Orbital Causality and the Geometric Problem

The anomalous perihelion precession of Mercury is the historical benchmark on which modern gravitational field theory was founded. Laplace [1] observed that if gravity were modelled as an attractive central force propagating at finite speed c , the retardation delay between source and target would produce an orbital aberration angle. This retarded force would exert a persistent transverse torque along the planet’s orbital velocity, causing orbital radii to expand and destabilizing planetary systems within thousands of years. Newtonian gravitation avoided this instability by postulating an instantaneous interaction ($c = \infty$). Modern relativistic field theories resolve the paradox through velocity-dependent interactions, which cancel the aberrant torque to $\mathcal{O}(v^3/c^3)$ [5].

Einstein [2, 3] and Schwarzschild [4] replaced the concept of a propagating mechanical force with the pseudo-Riemannian curvature of a four-dimensional spacetime manifold:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}, \quad (1)$$

so that an orbiting planet traverses a force-free geodesic through a pre-existing geometric landscape. General Relativity thus contains two kinds of entity: matter, described by its stress-energy, and geometry, described by the metric.

The objective of this paper is not to re-derive Eq. (1). Rather, I show that the verified empirical phenomena attributed to manifold curvature emerge from three short axioms on an uncurved Minkowski background, with momentum flux as the only kind of entity. I treat the problem as an if-then proposition: *if* matter consists of localized, non-superimposing circulations of invariant constituent speed c , whose momentum flux is additive and conserved, *then* operational rods and clocks immersed in the resulting ambient momentum sea must register effective metric intervals that match observation, without requiring empty space to act as a self-interacting geometric continuum.

Approaching gravitational physics through flat-space field interactions has an established pedigree spanning Thirring [6], Gupta [7], Feynman [8], Weinberg [9], and Deser [10]. The classical field-theoretic consistency theorems show that if one posits an independent, self-propagating massless spin-2 field in flat spacetime, demanding that it couple to a strictly conserved stress-energy tensor forces the field to couple recursively to its own energy-momentum, iteratively bootstrapping the non-linear Einstein–Hilbert action. That argument assumes that gravitation is an independent vacuum field with its own self-sourcing degrees of freedom. How the present framework obtains its second-order structure without that assumption is set out in Section 10.

The alternative I explore treats gravitation not as an independent vacuum field but as the emergent momentum exchange between extended matter assemblies. To describe an ordinary star like the Sun, General Relativity uses the exterior Schwarzschild metric and the interior Tolman–Oppenheimer–Volkoff (TOV) metric, two solutions of the same field equations joined across the stellar boundary. This matching is standard and well-behaved; the point is only that the present framework requires no matching step, because a single integral covers interior and exterior (Section 12.3). Under trapped surfaces and standard energy conditions, Penrose’s singularity theorems [11] prove that complete gravitational collapse in General Relativity terminates in geodesic incompleteness; on a flat substrate no geometric singularity can arise.

For the static single-centre tests through first post-Newtonian order and in the linearized sector, the framework’s predictions coincide with those of General Relativity, and its claims there are of explanatory economy (Section 15); multi-body interactions close via $N(N - 1)$ directed pairwise channels (demonstrated by closed triad chains), reproducing the post-Newtonian celestial baseline and guaranteeing exact bilateral action-reaction. Section 3 lists, for a reader

trained in General Relativity, each familiar expectation, how this paper handles it, and what agrees.

1.2 From Compton Phase to Celestial Trajectories

The formulation I develop here is geometric and kinematic. By postulating that matter is composed of localized internal speed- c motion, the framework establishes a causal bridge spanning more than twenty orders of magnitude, from the phase circulation of an atomic constituent to the orbit of Mercury.

At the microscopic level, rest inertia is the mechanical manifestation of internal speed- c circulation whose phase transit period defines the local cadence of proper time:

$$\Delta\tau = \oint_{\text{topology}} \frac{dl}{c}, \quad \hbar\omega_0 = m_0c^2. \quad (2)$$

When immersed in an ambient debt ϵ (equal to U/c^2 , the Newtonian potential over c^2 , in the weak field), the operational cadence scales as $d\tau/dt = \sqrt{1-2\epsilon}$ at first order (Section 10.9). Identifying the particle's internal quantum phase frequency with this operational clock rate shifts the effective constituent energy:

$$\hbar\omega(r) = \hbar\omega_0 \frac{d\tau}{dt} = m_0c^2 \sqrt{1-2\epsilon(r)} \approx m_0c^2 - \frac{GMm_0}{r}, \quad (3)$$

so that the ambient debt acts as the effective potential energy $V_{\text{eff}} = -m_0c^2\epsilon = -GMm_0/r$ in the non-relativistic Schrödinger equation:

$$i\hbar \frac{\partial \Psi}{\partial t} = \left(-\frac{\hbar^2}{2m_0} \nabla^2 + V_{\text{eff}} \right) \Psi, \quad (4)$$

in agreement with Colella–Overhauser–Werner (COW) neutron interferometry [57].

Along an arbitrary physical path between spacetime events A and B , the total phase accumulated by the constituent's internal circulation is:

$$\phi_{\text{accum}} = \int_A^B \omega(r) dt = \frac{m_0c^2}{\hbar} \int_A^B d\tau. \quad (5)$$

In wave mechanics (Feynman path integrals and Fermat's principle), physical propagation paths are determined by the condition of **stationary phase**:

$$\delta\phi_{\text{accum}} = 0 \iff \delta \int_A^B d\tau = 0 \iff \delta \left(-m_0c \int_A^B \sqrt{-\tilde{g}_{\mu\nu} \dot{x}^\mu \dot{x}^\nu} dt \right) = 0. \quad (6)$$

Here $\tilde{g}_{\mu\nu}$ is the effective metric read out by rods and clocks. The relativistic particle action and geodesic motion therefore follow from the phase interference of each constituent's internal speed- c circulation. Because m_0 drops out of the stationarity condition, every circulation follows the same path in a given debt field, whatever its composition. Attraction is therefore not a pull: bodies drift toward the region where their clocks run slow.

1.3 Architecture: Two Branches and One Operator

The framework develops from Axioms I–III in two branches:

- **Branch A (the Green's function, Sections 4–9).** All of projection and the coupling half of reception: how a confined source projects a stream, how the stream propagates, and what it deposits at a target. Integrated over an extended body, it gives the exact monopole drift and the exact arrival geometry.

- **Branch B (composition and relaxation, Sections 10–12).** The rules for composing Branch A objects into macroscopic bodies, and the relaxation of each circulation's cadence and extent under the debt it receives. Branch B adds no physics of its own: every conclusion it reaches must come out as a sum of Branch A objects.

Section 8 shows that at first order the two branches are two maps of one territory, joined by an exact dictionary that the source distribution generates itself.

1.4 Epistemic Scope and Domain Demarcation

1. **Kinematics of motion vs. particle taxonomy.** The scope of this paper is geometric and kinematic. The kinematics requires only a small set of class properties of matter (Section 11); which particle has which internal structure, and why, is not needed and is not addressed.
2. **Relational matter-on-matter measurement.** All physical observation is an interaction between material instruments:
 - *Immersion constraint.* When a target probe enters an ambient flux gradient, its internal speed- c budget is taxed, altering its proper clock cadence and physical rod scale.
 - *Finite target scale for clocks.* An operational clock requires a non-zero round-trip transit (Eq. 2); an idealized mathematical point has no internal transit time and can measure no proper time intervals.
 - *Open null topologies (photons).* Radiation has an open null topology without an internal rest frame; photons deflect not through internal clock dilation but because their wavevectors refract across the light cone via Fermat's principle. Because open null topologies complete no closed circulation transits, they possess identically zero singular sagitta residual and project no reciprocal debt field back onto the deflecting mass. A photon experiences no contact scattering or billiard-ball momentum exchange with the central body; rather, it refracts kinematically by flowing through the pre-existing ambient momentum-flux gradient ($n \approx 1 + 2\epsilon$) sustained by the source. Local momentum flux conservation ($\partial_\mu \Pi_{\text{total}}^{\mu\nu} = 0$) is satisfied everywhere along its stationary-phase trajectory without requiring a mechanical recoil on the distant emitter.
3. **The single-centre empirical baseline and system boundary.** The Solar System is the primary laboratory where relativistic gravitational mechanics is verified directly. Mercury's 43'' residual is extracted from ephemerides that integrate the multi-body post-Newtonian equations, and lunar laser ranging tests multi-body dynamics directly. In physical reality there is always more flux than can be accounted for; all physical models choose an operational boundary with appropriate truncation margins. Because distant isotropic flux linearly superposes into the ambient sea prior to instrument readout, it merely rescales local base units without inducing directional shears. Multi-body interactions close via $N(N - 1)$ directed pairwise channels (demonstrated by closed triad chains), guaranteeing exact bilateral conservation and reproducing the 1PN celestial baseline. Because the framework coincides with General Relativity at first order (including additive multi-source lensing, Section 14.4), extragalactic inferences drawn from lensing carry over unchanged; galactic and cosmological dynamics are outside the present scope.
4. **Rotation and radiation.** The explicit trajectory integration is restricted to the static, spherically symmetric exterior. In a rotating body, partially aligned constituent circulation adds an azimuthal momentum flux and a gravitomagnetic coupling $h_{0\phi} \propto J/r^2$, responsible for Lense–Thirring frame dragging [56]; the moving-source kernel fixes this term (Section 5.5). Accelerated sources radiate transverse speed- c momentum ripples;

at first order these carry only the two tensor polarizations, with no monopole or dipole component (Section 10.8). The radiated power is future work.

Free-streaming radiation propagates without completing a circulation transit and sources no debt; radiation confined within a body is part of that body's confined flux. Because the debt is retarded, an observer beyond outgoing radiation still registers the source's prior mass, while one inside it registers the reduced mass. In General Relativity, nonlinear gravitational-wave memory arises from the wave's own energy [25]; since free-streaming flux sources no debt here, that contribution may be absent or altered, and its form belongs to the radiation sector (Section 16).

5. **Interior regularity.** The derivations hold for mass distributions of finite central density. At first order the readouts are finite and positive throughout ordinary bodies; the areal chart is regular wherever $\zeta < 1$ (Section 12.3). For the Sun, $\epsilon(0) \approx 10^{-5}$ and $\epsilon(R_\odot) \approx 2.12 \times 10^{-6}$.
6. **Ultra-compact sources.** The strong-field exterior depends on how the response is completed beyond first post-Newtonian order. The exponential response of Section 10.3 gives no horizon and a photon sphere at $r \approx 3.30 GM/c^2$; these consequences are stated as conjectures (Section 13.4).

1.5 Relation to Prior Work

Several earlier programmes share elements of the present one. Optical-medium and polarizable-vacuum models (Eddington [16], de Felice [33], Puthoff [34]) also obtain $\gamma = 1$ from the response of clocks and rods on a flat background; they posit a medium property, whereas here the response follows from circulating matter. Whitehead's theory [14] coincides with the present single-centre readout at first post-Newtonian order and failed through metric superposition [17, 18]; here, momentum flux is linearly unified into an ambient sea prior to instrument readout, so isotropic backgrounds simply rescale base units without leaving residual directional dyad shears, identically yielding $\xi = 0$. Rosen's bimetric theory [26] failed through dipole radiation [27]; Logunov's relativistic theory of gravitation [28] retains a flat background with a dynamical gravitational field. Ni's stratified theory [29] uses the exponential static metric that appears here as the relaxation response, built from a scalar field on flat spacetime, and failed preferred-frame tests; Yilmaz's [30] and Puthoff's frameworks share the exponential static readout. Yilmaz's many-body dynamics were criticized by Misner [31], whose analysis turns on Yilmaz's treatment of field stress-energy as a gravitational source; in the present framework the field sources no debt, and many-body interactions close via $N(N - 1)$ directed pairwise channels (demonstrated by closed triad chains), guaranteeing exact bilateral conservation. Le Sage-type deficit pictures [35] share the idea of gravity as a deficit, but rely on absorption of a ballistic flux, which the balanced, non-ballistic generators of Axiom II avoid. The reading of the flat substrate adopted here, a physical arena whose rest frame is dynamically inaccessible because rods and clocks are built from the same flux, is close to the dynamical approach to relativity defended by Brown [46].

On the matter side, the picture of particles as stable, localized structures of a continuous relativistic field has a mainstream lineage: Skyrme's baryons as topological solitons [37], identified with large- N_c QCD baryons by Witten [38]; knotted solitons in the Faddeev-Niemi model [39], whose simplest stable form is a toroid, with trefoils at higher charge [40]; knotted solutions of Maxwell's equations [41]; spin as the angular momentum of circulating energy flow [42]; and zitterbewegung models of the electron as internal speed- c circulation [36]. The measured pressure distribution inside the proton, outward in the core and confining toward the periphery and integrating to zero [43], is the internal stress balance that Section 11 requires of every persistent structure.

The present framework is distinguished by its ontology (one kind of entity, with the debt as the residual of closed circulation), its criterion that only confined flux sources debt, its derivation of the exterior monopole by direct integration, its two-level sagitta, and its conservation axiom combined with the absence of self-sourcing. Table 1 lists the ways earlier flat-space theories failed and the present framework’s status against each.

Table 1: Known Failure Modes of Flat-Space Theories and Status in This Framework.

Failure mode	Theory eliminated	Status here
No light deflection ($\gamma = -1$)	Nordström [13]	Avoided: null streams of momentum flux; $\gamma = 1$
Direction-dependent $G(\xi)$	Whitehead [17, 18]	Avoided: flux is linearly unified prior to instrument readout; uniform backgrounds act as unit rescalings ($\xi = 0$)
Preferred frame (α_1, α_2)	Ni and stratified theories [29]	Avoided at linear order: covariant kernel
Light and gravity on different cones	Theories with distinct gravitational and light speeds	Absent: all unconfined flux shares the local light cone, consistent with GW170817 [48]
Laplace aberration	Retarded central forces [1]	Absent for uniformly moving sources (Section 5.5); acceleration terms belong to radiation
Dipole radiation	Rosen [26, 27]	Absent at leading order (conservation of momentum)
Scalar (breathing) wave modes	Scalar theories	Absent: tensor polarizations only; spherical parity and local flux conservation forbid breathing radiation
Non-conservation / bootstrap	Whitehead; Deser argument [10, 18]	Conservation is an axiom; second order from relaxation, not field self-sourcing (Section 10)
Many-body inconsistency	Yilmaz [31]	Resolved: $N(N - 1)$ directed pairwise channels enforce exact bilateral conservation ($\mathbf{F}_{ab} = -\mathbf{F}_{ba}$); field sources no debt
Horizonless strong field	Exponential-metric theories [32]	Physical demarcation: exponential response yields asymptotic compression and slowing down without event horizons or singularities (Sections 13.4, 16.2)

2 Axiomatic Foundations

I keep the axioms short. Anything that follows from geometry or conservation is derived in the main text or in Appendix A, not assumed.

Axiom I (Flat substrate). The arena of interaction is flat Minkowski spacetime $(\mathbb{R}^{1,3}, \eta_{\mu\nu})$, signature $\eta_{\mu\nu} = \text{diag}(-1, +1, +1, +1)$, with invariant speed $c < \infty$ governing all causal propagation. Its coordinates $(t, \mathbf{x})$ are static Euclidean charts that do not bend or warp. The substrate is physical: differences between the substrate chart and an instrument readout are physical displacements, not choices of description.

Axiom II (Matter as confined momentum flux). Momentum flux is the only kind of entity, and all of it propagates at speed c . Matter consists of localized, non-point regions of confined (trapped) speed- c momentum flux; the ambient sea is the superposition of the flux of all matter. Every circulation shares one speed budget, which in the absence of debt reads

$$c^2 = c_{\text{circ}}^2 + v^2, \quad (7)$$

where c_{circ} is the circulation’s internal speed in the substrate frame and v its translational speed. Only confined flux, flux that completes a circulation transit, sources debt; free-streaming flux

does not. This holds at every scale: a bound system, such as a nucleus, a star or an orbiting pair, is itself a confining assembly, so its confined flux, including the interaction and binding flux of its constituents, sources debt. Covariantly, flux is confined when it belongs to a bound subsystem whose total 4-momentum is timelike: light in a cavity is confined together with its cavity, photons diffusing out of a star are confined until they escape, and a photon passing a star is free. Free light therefore does not gravitate: free-streaming photons surf ambient debt gradients without sourcing a reciprocal debt field back onto the deflecting mass.

Axiom III (Additivity, conservation, relative debt, order of operations). Momentum flux is additive: in transit, contributions superpose linearly and do not scatter off one another. All flux, confined or free, including the projected streams of Section 5, propagates at the local speed set by the unified debt: each confined source multiplies the local cadence and propagation of everything passing through by $e^{-\epsilon_j}$, factors compose by multiplication, and debts therefore add. The total momentum flux of matter and propagating flux is locally conserved, $\partial_\mu \Pi_{\text{total}}^{\mu\nu} = 0$; in this sense the substrate's flux is divergence-free in spacetime. Only debt relative to the local ambient level is accessible: uniform contributions are a change of units. The distribution is unified first into an ambient sea and read out once: effective geometry is a downstream readout of rods and clocks made of the same flux and holds no momentum, so it is never superposed, locally patched, or treated as a dynamical field. Every modification to the physical scenario—including the introduction, displacement, or removal of any constituent—demands a complete reassessment of the full $N(N-1)$ directed pairwise relational channels across the entire system prior to instrument readout. Interactions between pairs do not dilute with total constituent count (N); each directed pair interacts via an invariant one-to-one momentum flux exchange. All multi-body interactions close at the pairwise level, which guarantees exact bilateral action-reaction ($\mathbf{F}_{ab} = -\mathbf{F}_{ba}$) and global conservation, with higher-order groupings forming chains of directed pairs.

Consequences recorded here. Eq. (7) gives special-relativistic time dilation directly: proper time is internal transit, so $d\tau/dt = c_{\text{circ}}/c = \sqrt{1-v^2/c^2}$. The equivalence of inertial and passive mass follows from the universal confinement response together with persistence and conservation (Section 11); the universal active-to-passive ratio follows from conservation (action equals reaction); the absence of a scalar wave mode follows from conservation of the source (Section 10.8). Because light and propagating debt obey the same local speed, gravitational ripples and light share one light cone, as the near-simultaneous arrival of gravitational waves and gamma rays from GW170817 requires [48].

2.1 Stated Rules and Calibrations

Beyond the axioms, the paper uses two stated rules and one calibration, and nothing else.

1. **Projection rule (the Green's function).** A confined source projects a retarded null stream whose deposit at a field point is its relaxed energy times the arrival dyad over the retarded distance (Section 5). If the propagating influence obeys a local, linear (additive) law at speed c in three dimensions with no incoming flux, its retarded Green's function is fixed, $\delta(t-L/c)/L$; the $1/L$ profile, which is the arc curvature of the sagitta, therefore follows from Axioms I and III. The null dyad $\ell \otimes \ell$ follows because the flux moves at c . Streams propagate at the local speed of light (Axiom III); at first order the debt profile is unchanged, and the local speed enters timing and second-order cross terms. What remains as input is which flux sources debt (Axiom II) and the magnitude, G .
2. **Response rule (relaxation).** A confined circulation responds to an increment of debt the same way in its own local units, whatever background it already sits in. With relative debt

(Axiom III), this fixes the exponential response (Section 10.3).

3. **Calibration.** Newton's constant G , as in General Relativity. The first-order clock coefficient $C = 2$ is not an empirical calibration, but the exact kinematic derivative of the quadratic speed budget ($c^2 = c_{\text{circ}}^2 + v^2$) and rod-clock duality (Sections 10.3, 10.9, and 12).

Deriving G from the structure of matter is open (Section 16).

3 Reader's Map: From Expectations to This Framework

A reader trained in General Relativity will bring well-founded expectations. Table 2 lists the principal ones, how this paper handles each and why, and what agrees with General Relativity. The last column, read alone, is the paper's statement of empirical equivalence through first post-Newtonian order. Table 3 anchors each term used here to its nearest counterpart in General Relativity.

Table 2: Expectations of a Reader Trained in General Relativity, Converted.

A reader expects	In this paper	Why	Agrees with GR on
Gravity is curvature set by a field equation	A debt on a flat substrate; effective geometry is what instruments built from flux read out	Axioms I, III	Exterior Schwarzschild through 1PN: 43'', 1.75'', Shapiro delay
The field equation comes first	The Green's function comes first; summed over constituents it should give the balance law (to be verified)	Additivity	Linearized GR in harmonic gauge
The shell theorem and Birkhoff are used	Harmonic volume averaging yields static monopole drift; point-mass reduction is invalid for T_{ij} , breaking Birkhoff via $\langle r'^2 \rangle$	Direct integration and parity	Monopole matches GR; arrival tensor retains interior memory
Coordinate differences are gauge	The term $\partial_i \partial_j \Xi$ is a physical displacement between substrate and readout, unseen by instruments at first order	Axiom I	Every first-order observable
Nonlinearity comes from field energy gravitating	Second order comes from sources relaxing, applied causally; binding is counted once	Sec. 10	$\beta = 1$
Bodies are point particles on geodesics	Bodies are extended circulations; cycle-averaged they move as points	Laue's theorem (Sec. 11)	Geodesic motion
The equivalence principle is postulated	It follows from the universal confinement response, a postulate natural to the ontology, with persistence and conservation	Sec. 11	Universality of free fall
Post-Newtonian theory runs to arbitrary order	Continuous sagitta mechanism and one exact response; expansion in ϵ is for comparison only	Sec. 6, 10	PPN $\gamma, \beta; \alpha_1, \alpha_2$ at linear order
Self-force needs regularisation	There are no points; effective self-force is coarse-graining; radiation reaction is emission recoil	Sec. 10.7	Quadrupole radiation (to compute)
Preferred frames are excluded by covariance	The kernel is covariant; uniformly moving sources show no aberration	Sec. 5.5	$\alpha_1 = \alpha_2 = 0$ at linear order
G is fundamental	G is a calibration	As in GR	Same
A new theory must change Solar System predictions	Through 1PN it does not; multi-body dynamics close via pairwise triad chains ($N(N-1)$ directed channels); differences are confined to near-limb Region 2 and ultra-compact objects	Sec. 9, 16	Empirical baseline matches GR across all tested regimes
Gravitational waves travel at c	All unconfined flux shares the local light cone	Axiom III	GW170817 timing

Table 3: Glossary with Counterparts in General Relativity.

Term	Meaning here	Nearest GR counterpart
Debt, ϵ	Cadence deficit relative to the ambient level	U/c^2 in the weak field
Singular sagitta (Generator)	Residual of one circulation's source-centred arcs, curvature $1/L$	Retarded Green's function kernel ($1/r$)
Relational sagitta (Pairwise)	Angular interference of two sources seen from a target ($1 - \cos \theta_{bc}$)	Transverse hoop stress and arrival tensor structure
Arrival tensor, T_{ij}	Summed $\hat{n}_i \hat{n}_j / L$ at a point, equal to $\epsilon \delta_{ij} - \partial_i \partial_j \Xi$	Spatial part of the linearized metric plus a coordinate term
Branch A	Source and propagation: the Green's function	The field side
Branch B	Composition, relaxation and readout	The metric side
Relaxation	A circulation adjusting its cadence and extent to the debt it receives	Clock and rod response to the metric
Fold-back	Reapplying the projection with relaxed sources	Iterating to a self-consistent static solution
Transit	One internal circulation cycle	A proper-time tick
Separatrix	Border between a knot's trapped flux and its escaping flux	A body's world-tube boundary
Regions 1, 2, 3	Far field ($r > \sqrt{2}R$), near limb ($R < r < \sqrt{2}R$), interior	Exterior and interior Schwarzschild

4 Valid and Invalid Cancellation

Every derivation in this paper sums contributions that partly cancel. Whether a cancellation is legitimate is therefore the first question, and I settle it once here. A cancellation is *valid* only if it passes all three tests of Table 4.

Table 4: The Three Tests for a Valid Cancellation.

Test	Valid	Invalid
Order	Each stream forms its own residual first, then streams combine (project, then sum)	Combining before the residual exists: averaging to a centroid, or adding one circulation's two arms first (sum, then project)
What cancels	Odd, directional quantities: delivered momentum, such as the pushes of opposite limbs	Even quantities treated as odd: the sagitta, or the spread of arrival directions
Ledger	The cancelled flux reappears as stress (transverse, or "hoop", flux), and the conservation law balances	The cancelled flux leaves no trace; information is destroyed

Applied to the results that follow:

- **Valid:** Harmonic volume averaging of $1/L$ for the odd-parity net drift vector **N** (1st moment; Section 7); the net flux falling to zero at the centre, where the cancelled part becomes an isotropic bath; the opposition of pairs of streams within $\sqrt{2}$ body radii, stored as transverse hoop stress (Section 9).
- **Invalid:** Collapsing an extended body of arbitrary shape to an idealized centroid point mass or pre-computing an intermediate "effective geometry" prior to unifying the flux. Point-mass reduction artificially sets constituent curl scale $s \rightarrow 0$ and aperture $\Theta \rightarrow 0$, destroying transverse hoop stress B and eliminating Region 2; similarly, superposing effective metrics rather than unifying raw momentum flux violates Axiom III and produces spurious directional shears.

The shell theorem holds strictly and exclusively for the harmonic volume average of the scalar Green's function in spherical symmetry; treating flowing momentum flux as if it originated from a point mass is an invalid cancellation that destroys physical geometric information, and arbitrary non-spherical bodies must be integrated directly as distributed flux.

5 Branch A: The Green's Function

A microscopic circulation has no gravitational manifestation in total isolation; it becomes operative through reception by another circulation. The Green's function of the framework describes one source constituent acting on one field point. It contains all of projection and the coupling half of reception, and nothing of the target's own response, which belongs to relaxation (Section 10).

5.1 Source-Centred Arcs and the Singular Sagitta

Consider a source constituent of energy E and a target at distance L . The source's confined flux has non-vanishing internal curl. Seen from the target, a circulation's flux projects as two equal and opposite arc sweeps centred on the source, with radius of curvature L :

1. *Sideways cancellation.* The two sweeps run in opposite transverse directions, so their sideways components cancel, $\mathbf{s}_{(+)}^\perp + \mathbf{s}_{(-)}^\perp = \mathbf{0}$. This odd-parity cancellation ensures that the source exerts no net transverse thrust or ballistic pressure on the target.
2. *Constructive inward curvature.* Both arms curve inward, away from their tangent lines, toward the source. For an arm of reach s subtending $\theta = s/L$, the sagitta, the depth from chord to arc, is

$$h = L(1 - \cos \theta) \approx \frac{1}{2}L\theta^2 = \frac{s^2}{2L}, \quad (8)$$

pointing toward the source. Sagittas are positive-definite and add, $h_{(+)} + h_{(-)} = s^2/L > 0$.

Here s is the scale of a single circulation, the Compton scale for an elementary constituent, so the aperture s/L is microscopic at any macroscopic distance. The quantity that survives every size is the arc's *curvature*,

$$\kappa = \frac{2h}{s^2} = \frac{1}{L}, \quad (9)$$

which is the same for every circulation. This is the *singular sagitta*: one circulation's residual, seen from one field point. Its $1/L$ profile is also the retarded Green's function of any local, additive law propagating at c in three dimensions (Section 2.1), so the sagitta construction and additivity give the same kernel.

5.2 The Kernel and Its Three Parts

For a source constituent j with 4-velocity u_j and retarded null separation R_j from its retarded position to the field point x , I define the retarded Green's propagator kernel

$$\mathcal{K}_{\mu\nu}(x \leftarrow j) = \frac{2G E_j^{\text{rel}}}{c^4} \frac{\ell_\mu^{(j)} \ell_\nu^{(j)}}{\rho_j}, \quad \rho_j = -u_j \cdot R_j, \quad \ell^{(j)} = \frac{R_j}{\rho_j}. \quad (10)$$

In the source's rest frame $\rho_j = L_j$ and $\ell^{(j)} = (1, \hat{\mathbf{n}}_j)$, with $\hat{\mathbf{n}}_j$ the arrival direction. The kernel has three parts:

1. **Source half.** Emission from the source's relaxed state: its energy E_j^{rel} , set by the ambient debt at its own location, and its own motion through u_j .

2. **Propagation.** Retarded and null with respect to the local light cone, diluted with arc curvature $1/\rho_j$; at first order the substrate form gives the debt profile, and the local speed enters timing and second-order cross terms.
3. **Coupling half of reception.** What the stream deposits at the target: a debt increment along its arrival direction, $\ell \otimes \ell$. This is what a null stream of momentum flux is, so the dyad is part of the Green's function, not a readout assumption.

The target's own transformation, how its cadence and extent change while it keeps local light speed at c , is excluded; it is relaxation. Swapping source and target swaps the content, not the form, so action equals reaction by construction.

For a static source the time-time component gives the debt,

$$\epsilon_j(\mathbf{x}) = \frac{G}{c^4} E_j \kappa_j = \frac{G m_j}{c^2 L_j}, \quad (11)$$

that is, source energy times the curvature of its arcs at the target, divided by c^4/G . The debts of all sources add on the flat substrate, $\epsilon = \sum_j \epsilon_j$.

5.3 From the Sagitta to the Inverse-Square Drift

The step from the singular sagitta to Newton's law can be followed through one cycle of a source S and a target T . Numerical factors of order one from loop geometry are absorbed into the coupling calibration G .

1. **Projection.** Each transit of a source circulation of scale s_S leaves an inward residual of order $(E/c)(s_S/r)$, at a rate of order c/s_S transits per unit time. The product is E/r : the constituent's size cancels, and the arc curvature $1/r$ remains. The debt arrives at T after a delay r/c .
2. **Relaxation.** With the arriving field frozen, T 's circulation adjusts its cadence and extent (Section 10). Nothing moves.
3. **Reception.** T 's circulation spans the field: its near and far turning points, a distance L_T apart, see arc curvatures differing by $\Delta\kappa \approx L_T/r^2$. Flux turned back at the near point carries more momentum per unit energy than flux turned back at the far point, because the local index is higher there, and over a round trip of duration $2L_T/c$ the loop length cancels (Section 11.3). The force is

$$\mathbf{F} = \frac{G}{c^4} E_S E_T \nabla \kappa = -\frac{G(E_S/c^2)(E_T/c^2)}{r^2} \hat{\mathbf{r}}. \quad (12)$$

4. **Update.** T 's momentum changes; by the same construction with the roles swapped, S drifts toward T with an equal and opposite force. During the light-travel delay the difference is carried by the propagating flux.

Equivalently, by stationary phase (Eq. 6) with cadence $d\tau/dt = \sqrt{1-2\epsilon}$,

$$\mathbf{a} = -c^2 \nabla \left(\frac{d\tau}{dt} \right) \approx c^2 \nabla \epsilon = -\frac{Gm}{r^2} \hat{\mathbf{r}}. \quad (13)$$

Two scales cancel, the source's constituent size in projection and the target's loop length in reception; the geometry that survives is the curvature field and its gradient. Energy is the multiplier and geometry the mechanism. Corrections of order $(s_S/r)^2$ and $(L_T/r)^2$ are kept explicit in principle and are negligible at any macroscopic distance.

5.4 Tangent-Space Descriptions and the Order of Summation

When the conventional stress–energy tensor $T^{\mu\nu}$ is constructed, internal circulation is mapped onto tangent bivectors $B^{\mu\nu} \in \Lambda^2(T_p M)$. In the differential limit $\theta \rightarrow 0$ the sagitta vanishes to second order relative to the chord,

$$\lim_{\theta \rightarrow 0} \frac{h}{s} = \lim_{\theta \rightarrow 0} \frac{\frac{1}{2}L\theta^2}{L\theta} = 0, \quad (14)$$

so equal and opposite circulating elements cancel point-wise. Because projection and summation do not commute across distributed sources,

$$\sum_k \text{Proj}(\mathbf{a}_k) \neq \text{Proj}\left(\sum_k \mathbf{a}_k\right), \quad (15)$$

the order matters. The stress–energy tensor accounts for the energy and stresses of circulation, but in tangent-space form it does not expose the finite sagitta geometry. Stokes’ theorem holds for finite loops as well as infinitesimal ones; the distinction concerns the order of operations, not loop size. This framework forms each constituent’s sagitta first and sums afterwards (Table 4).

5.5 Moving Sources: No Aberration

For a source in uniform motion with velocity $\mathbf{v}$, the retarded distance in Eq. (10) depends only on the source’s *present* position. With $\mathbf{d} = \mathbf{x} - \mathbf{x}_{\text{present}}(t)$,

$$\rho = \gamma \sqrt{d^2 - |\mathbf{v} \times \mathbf{d}|^2 / c^2} \quad (16)$$

(Appendix A.7; checked numerically against the retarded-time solution to ten digits). Retardation therefore Lorentz-contracts the debt about the present position and produces no lag toward the retarded position: Laplace’s aberration is absent for uniformly moving constituents. The moving kernel is the boost of the static one, which fixes the gravitomagnetic part, and hence frame dragging, with the coefficient of linearized General Relativity. What remains depends on acceleration, enters at $\mathcal{O}(v^3/c^3)$, and belongs to radiation, the same structure Carlip identifies in General Relativity [5].

5.6 Synchronization of an Extended Source

The time–space part of the summed kernel for static sources is an exact gradient, for any distribution:

$$h_{0i} = -\frac{2G}{c^4} \int dE \frac{\hat{n}_i}{L} = -\partial_i \Lambda, \quad \Lambda = \frac{2G}{c^4} \int dE \ln L. \quad (17)$$

A single resynchronization of clocks removes it everywhere, inside and outside matter.

5.7 The Radiation Ledger

A static or uniformly moving constituent is a balanced generator and debits no momentum at projection. An accelerating source radiates: the escaping flux carries net energy and momentum, and the momentum ledger must debit the source at projection by the corresponding emission recoil. This is how orbital decay arises without any circulation acting on itself (Section 10.7).

6 The Unified Sagitta Mechanism: Microscopic Generator and Relational Pairwise Ledger

The sagitta operates not as two separate physical laws or sequential computational stages, but as one continuous geometric mechanism: the microscopic singular sagitta of individual circulations generates the retarded Green's function, while the relational sagitta of constituent pairs expresses the angular interference of arriving flux streams across the directed channels of an extended assembly.

Table 5: The Unified Sagitta: Microscopic Generator and Relational Angular Geometry.

	Microscopic Generator (Singular Sagitta)	Relational Geometry (Pairwise Sagitta)
Elements	Two arms of one circulation (chord); field point (vertex)	Two arriving source streams b, c (chord); target a (vertex)
Aperture at target	Circulation scale over distance (s/L): microscopic	Angle θ_{bc} subtended at target: up to full field of view Θ
Its $\sqrt{2}$ boundary	$\sqrt{2}$ times circulation scale (purely intra-constituent)	$\sqrt{2}$ times body radius (onset of opposing limb pairs)
Order in the flux	Linear: defines the Green's function kernel $1/L$	Bilinear: pairwise angular interference $w_b w_c (1 - \cos \theta_{bc})$
Governs	Retarded null stream projection and scalar debt ϵ	Arrival tensor T_{ij} , hoop stress B , Region 2 dynamic softening

6.1 Microscopic Generator: The Singular Sagitta

At the microscopic foundation, Compton-scale circulations disperse flux into the ambient sea. Each constituent circulation j possesses non-vanishing symmetric curl. When viewed from another constituent k at distance L_{jk} , the failure of the curl arms to cancel to a tangent-space point leaves a residual inward arc curvature,

$$\kappa_{jk} = \frac{1}{L_{jk}}. \quad (18)$$

Because every stream is far-field with respect to its own microscopic circulation scale ($s \ll L$), the singular sagitta generates the exact monopole drift down to the macroscopic surface of an extended body, even when the body fills half the target's sky.

6.2 The Directed Relational Ledger and Newton's Third Law

A persistent source of confusion in multi-body relational bookkeeping is the conflation of scalar binding energy with directed momentum flux. In the physical momentum ledger on the flat Minkowski substrate, momentum transfer is inherently directed and causal:

1. **Directed Projection:** Constituent j acts as a source, projecting a retarded null stream to target k . The resulting momentum debt gradient imparts an inward drift acceleration on target k :

$$\mathbf{a}_{k \leftarrow j} = c^2 \nabla_k \epsilon_j(\mathbf{x}_k) = -\frac{Gm_j}{L_{jk}^2} \hat{\mathbf{n}}_{jk}. \quad (19)$$

2. **Bilateral Reciprocity:** Simultaneously, target k acts as a source, projecting a retarded stream back to constituent j :

$$\mathbf{a}_{j \leftarrow k} = c^2 \nabla_j \epsilon_k(\mathbf{x}_j) = -\frac{Gm_k}{L_{kj}^2} \hat{\mathbf{n}}_{kj} = +\frac{Gm_k}{L_{jk}^2} \hat{\mathbf{n}}_{jk}. \quad (20)$$

For an assembly of N constituents, the interaction ledger consists of all directed source-to-target channels:

$$\mathcal{C}_{\text{directed}} = \sum_{j=1}^N \sum_{k \neq j}^N (j \rightarrow k). \quad (21)$$

There are exactly $N(N - 1)$ directed channels:

- For $N = 2$ (a binary pair), there is 1 physical pair, but $2(1) = 2$ directed channels ($\mathbf{a}_{2 \leftarrow 1}$ and $\mathbf{a}_{1 \leftarrow 2}$). Evaluating each directed projection once directly manifests Newton's third law ($\mathbf{F}_{12} = -\mathbf{F}_{21}$).
- For $N = 3$ (a triad), there are 3 physical pairs, yielding $3(2) = 6$ directed channels, forming a closed triad of mutual momentum transfers.
- For general N , the full sum over all $N(N - 1)$ directed channels evaluates both physical halves of bilateral momentum conservation across all pairs.

No halving of momentum transfers. Summing the directed momentum transfers does not double-count; it accounts for both directions of physical momentum flux crossing the substrate at speed c . Halving the directed momentum ledger would artificially extinguish one direction of Newton's third law. The familiar factor of $1/2$ belongs strictly and exclusively to undirected scalar quantities, such as the total gravitational binding energy:

$$\mathcal{E}_{\text{bind}} = -\frac{1}{2} \sum_{j=1}^N \sum_{k \neq j}^N \frac{Gm_j m_k}{r_{jk}} = -\sum_{j < k} \frac{Gm_j m_k}{r_{jk}}. \quad (22)$$

In scalar potential energy bookkeeping, $\mathcal{E}_{\text{bind}}$ is a single scalar state property assigned to the bound system as a whole; the factor of $1/2$ converts the sum over ordered pairs (j, k) into a sum over unordered physical bonds $\{j, k\}$. Vector momentum $\mathbf{p}$, by contrast, is localized, directed, and physically transferred in both directions.

No connectivity dilution. There is no division of the interaction by the constituent count $(1/N)$ or channel count. Gravity does not dilute as connectivity increases; each source-target pair constitutes an independent, invariant speed- c momentum exchange channel. Divergence is prevented not by ad-hoc dilution factors, but by the universal speed budget ($c^2 = c_{\text{circ}}^2 + v^2$) and the universal exponential relaxation ($e^{-\epsilon}$).

6.3 The Arriving Angular Identity at a Target

When an external target a observes an extended assembly of sources $\{b\}$, each stream arrives with weight w_b along unit direction $\hat{\mathbf{n}}_b$. The scalar flux is $I = \sum_b w_b$, and the net vector flux is $\mathbf{N} = \sum_b w_b \hat{\mathbf{n}}_b$. Along any single stream, absolute minus net along the arrival direction gives the sagitta factor $1 - \cos \theta$. Summed over all pairs of arriving streams, we obtain the exact algebraic identity:

$$I^2 - |\mathbf{N}|^2 = \sum_{b=1}^N \sum_{c=1}^N w_b w_c (1 - \cos \theta_{bc}). \quad (23)$$

The diagonal terms ($b = c$) vanish identically because $\theta_{bb} = 0 \implies 1 - \cos(0) = 0$. The off-diagonal terms span all $N(N - 1)$ ordered pairs of arriving streams. This full sum is the exact measure of directional opposition and transverse hoop stress (B , Section 7); it is *not* halved, as the full quantity $I^2 - |\mathbf{N}|^2$ directly represents the scalar-to-vector flux deficit.

6.4 Closure at Directed Pairs: The Triad as Multi-Body Demonstration

Why does relational geometry close strictly at directed pairs? The triad ($N = 3$) serves not as a distinct multi-body primitive, but as the minimal demonstration that multi-body gravitational mechanics requires no independent three-body potentials. In a triad of constituents $\{a, b, c\}$, there are exactly $3(2) = 6$ directed pairwise channels. Each constituent projects its singular Green's function stream along two directed lines of sight and receives two incoming streams. The entire interaction network is fully determined by summing these six directed projections; all sagitta contributions are additive, yielding the net drift of the assembly while preserving exact bilateral action–reaction ($\mathbf{F}_{ab} = -\mathbf{F}_{ba}$).

Geometrically, a set of directions in three-dimensional Euclidean space is completely fixed, up to rigid rotation, by its pairwise dot products. Gram's determinant proves that the pairwise angles θ_{bc} exhaust all rotation-invariant 3D spatial geometry: no independent three-body or higher-order angular invariants exist. The relational sagitta ($a \leftarrow \{b, c\}$) is quadratic in arriving flux ($w_b w_c$), emerging algebraically from the square of the unified vector flux ($I^2 - |\mathbf{N}|^2$). Higher-order multi-body groupings (such as the Einstein–Infeld–Hoffmann equations of motion) are closed chains of directed pairwise relations ($a \leftrightarrow b, b \leftrightarrow c, c \leftrightarrow a$).

In accordance with Axiom III, the distribution is unified before readout, so instrument response never acts on individual streams, but on the unified aggregate. A single response rule is then applied exactly to the unified distribution; post-Newtonian expansion is used only for empirical comparison.

7 Direct Integration over an Extended Body

I now integrate the Green's function over a spherical body, shell by shell, to a target outside it. Neither the shell theorem nor Birkhoff's theorem is assumed.

7.1 The Scalar Debt: The Monopole as a Result

For a thin shell of radius a and a target at $r > a$, the shell average of $1/L$ is exactly $1/r$, because $1/L$ is harmonic (Lemma A.1). Summing shells,

$$\epsilon(r) = \frac{GM}{c^2 r} \quad (r \geq R), \quad (24)$$

for any spherical profile, down to the surface, and the net drift $c^2 \nabla \epsilon$ is the exact inverse-square monopole (Theorem A.2). Crucially, the inverse-square law is an exact mathematical consequence of harmonic volume averaging of the scalar Green's function ($1/L$) for the odd-parity first moment ($\mathbf{N}$); it does *not* imply that the extended body acts physically as an idealized point mass. Because total momentum flux is locally conserved ($\partial_\mu \Pi^{\mu\nu} = 0$) and spherical symmetry forbids net directional flux by parity, any radial pulsation of a spherically symmetric body leaves the total active mass and the exterior monopole strictly static.

7.2 The Arrival Tensor: Exact, with One Terminating Term

The direction-resolved sum of the static kernel is the arrival tensor,

$$T_{ij}(\mathbf{x}) = \frac{G}{c^4} \int dE \frac{\hat{n}_i \hat{n}_j}{L}. \quad (25)$$

The shell average of L itself is $r + a^2/3r$ (Lemma A.1): L is not harmonic, so a shell is not equivalent to a point. Summed over any spherical profile, with $\langle r'^2 \rangle$ the energy-weighted mean square

radius of the source,

$$T_{ij} = \frac{GE}{c^4} \left[\frac{\hat{r}_i \hat{r}_j}{r} - \frac{\langle r'^2 \rangle}{3r^3} (3\hat{r}_i \hat{r}_j - \delta_{ij}) \right] \quad (r \geq R) \quad (26)$$

(Theorem A.4). The series stops after one term. A direct integration with six million sample points agreed with Eq. (26) to four or five digits for both a uniform and a centrally concentrated profile, including at $r = 1.2R$.

The relative radial deficit is $\frac{2}{3}\langle r'^2 \rangle / r^2$, which is $\frac{2}{5}(R/r)^2$ for a uniform sphere. This is the pairwise sagitta correction of earlier versions of this work, now correctly booked: it is a transfer from radial to transverse arrival with the trace unchanged, not an extra radial force. At the surface of a solid uniform sphere the arrival tensor is radial $3/5$ and transverse $1/5$ (in units of $GM/c^2 R$); it is isotropic at the surface only for a thin shell.

7.3 Momentum Flux Is a Conserved Tensor

With flux weighting $1/L^2$, the summed momentum-flux tensor of the arriving streams has a radial part A and a transverse part B per transverse direction. Each point source's flux is divergence-free away from it, so the sum obeys the tensor conservation law (Appendix A.3)

$$A' + \frac{2(A - B)}{r} = 0. \quad (27)$$

For a point source, $B = 0$ and $A \propto 1/r^2$, the shell-theorem form. Near an extended body $B > 0$, so the radial flux falls short of the point-source value and the transverse (hoop) flux carries the difference, with momentum exactly conserved. Table 6 gives the computed values for a uniform sphere; the conservation law holds numerically to between 10^{-8} and 10^{-10} .

Table 6: Radial and Transverse Momentum Flux Outside a Uniform Sphere (Relative to a Point Source of Equal Energy).

r/R	1.1	1.2	1.5	2.0	3.0
Radial flux $A r^2$	0.806	0.842	0.904	0.948	0.977
Transverse flux $B r^2$	0.241	0.184	0.104	0.054	0.023

7.4 Which Moment the Shell Theorem Governs

The arriving flux at a point is a distribution over directions, and each result above is one of its moments (Table 7).

Table 7: Moments of the Arrival Distribution.

Moment	Parity	Cancels?	Governs
0th: absolute flux I	even	no	The count of arriving flux
1st: net flux $\mathbf{N}$	odd	yes, by conservation	The drift: exact monopole
2nd: arrival tensor $\hat{\mathbf{n}} \otimes \hat{\mathbf{n}}$	even	no	Geometry: the sagitta and the regions

The shell theorem is a theorem strictly about harmonic volume averaging of the odd-parity vector moment $\mathbf{N}$; it governs net momentum delivery and is completely blind to the scalar count I and to the second-moment arrival tensor T_{ij} . Treating flowing momentum flux as if

it originated from a collapsed centroid point mass is an invalid cancellation (Section 4) that destroys physical geometric information. The drift is the gradient of the trace of the residual, $\nabla(1/L) = -\hat{\mathbf{n}}/L^2$, so the Bernoulli form (gradient of a quadratic quantity) and the conservation form (net momentum delivered) are the same statement.

7.5 The Relational Departure from Birkhoff's Theorem and Metric Blindness

In General Relativity, Birkhoff's theorem declares that any spherically symmetric vacuum exterior is strictly static and described by the Schwarzschild metric. This establishes metric blindness: the vacuum geometry is completely independent of internal radial dynamics and density structure $\langle r'^2 \rangle$. In conventional field theory, this blindness is enforced by positing test particles as dimensionless mathematical points devoid of spatial extent or internal circulation.

In the present framework, where all matter consists of momentum flux, any physical probe capable of performing a readout is necessarily an extended assembly of circulating flux with a non-zero transit scale. While the isotropic first moment (the scalar drift and clock cadence) satisfies Birkhoff's theorem, the physical arrival geometry fundamentally breaks it:

1. **The net drift and scalar cadence satisfy Birkhoff's theorem.** Because total momentum flux is locally conserved ($\partial_\mu \Pi^{\mu\nu} = 0$) and spherical symmetry forbids net directional flux by parity, any radial pulsation of a spherically symmetric body leaves the total active mass M and the exterior monopole $\epsilon(r) = GM/c^2 r$ strictly invariant. Isotropic clock cadence and net drift acceleration are blind to internal radial pulsations.
2. **The arrival tensor T_{ij} explicitly breaks Birkhoff's theorem.** Unlike the harmonic potential $1/L$, the arrival tensor T_{ij} in Eq. (26) depends directly on the internal density profile through the energy-weighted mean-square radius $\langle r'^2 \rangle$. A pulsating star possesses an invariant scalar monopole, but an actively breathing arrival geometry.

Point-mass reduction is an invalid cancellation because it artificially sets constituent scale $s \rightarrow 0$ and aperture $\Theta \rightarrow 0$, destroying transverse hoop stress B and erasing Region 2. The arrival tensor T_{ij} , transverse hoop stress, and the $\sqrt{2}R$ limb boundary are entirely outside the reach of Birkhoff's theorem. Because real measuring instruments consist of internal circulations spanning non-zero aperture, they couple directly to the full arrival tensor T_{ij} rather than an idealized scalar point-metric. Where this geometric memory becomes dynamically operative is when bodies carrying bulk momentum traverse Region 2 (Section 9).

8 The Bridge between Branch A and Branch B

At first order, the sagitta map of Branch A and the rod-clock readout of Branch B are two maps of one territory, joined by an exact dictionary that the distribution generates itself, for any distribution.

Each stream satisfies $\hat{n}_i \hat{n}_j / L = \delta_{ij} / L - \partial_i \partial_j L$ exactly. Summed over any source distribution,

$$T_{ij} = \epsilon \delta_{ij} - \partial_i \partial_j \Xi, \quad \Xi = \frac{G}{c^4} \int dE L. \quad (28)$$

- $\epsilon \delta_{ij}$ is Branch B's scalar readout: clocks and rods responding to the unified debt.
- $\partial_i \partial_j \Xi$ is a displacement by $\nabla \Xi$ between substrate and readout: physical in the substrate (Axiom I), unseen at first order by instruments made of the same flux.
- The time-space term is the gradient of Λ (Eq. 17), removed by resynchronization.

The dictionary is built from two integrals of the Green's function, $\int dE L$ and $\int dE \ln L$; nothing is imported. A deliberately lopsided source, a triaxial ellipsoid with axes $1.0 \times 0.6 \times 0.3$ plus an

off-centre blob of radius 0.3 at half weight, satisfies Eq. (28) to 2×10^{-7} at three exterior points, the precision of the finite differences used. No symmetry is needed.

The translation runs one way. From Branch A to Branch B always works: forget Ξ . From Branch B to Branch A does not: two bodies with the same exterior debt can differ in $\langle r'^2 \rangle$, so their sagitta maps differ. Branch B is the forgetful image of Branch A, and the sagitta map is the richer description.

Precession bump vs. geometric aperture bump. A vital physical distinction must be maintained between the anomalous precession of Mercury and the geometric aperture bump of an extended body. Mercury's 43.0''/century precession is driven entirely by the coupling of its own *bulk orbital momentum* to the Sun's scalar debt gradient $\nabla\epsilon$ through the universal speed budget (Section 13.1); it is independent of the Sun's physical radius and would be identical if the Sun were a compact star. At Mercury's orbit ($r \approx 66 R_\odot$), the body-scale geometric bump $(R_\odot/r)^2 \sim 2 \times 10^{-4}$ is negligible, so the arrival tensor is purely radial to high precision. The geometric aperture bump becomes dynamic only when an extended body carrying bulk momentum enters Region 2 ($R < r < \sqrt{2}R$), where the wide-angle arrival tensor T_{ij} couples to translational velocity to produce dynamic softening (Section 9).

9 Three Regions

The exterior and interior of an extended body divide into three regions, defined by two different boundaries: $\sqrt{2}R$, where opposed pairs of streams first appear (a pairwise boundary), and the surface, where the net flux starts to fall (a linear boundary). Outside the body the drift is the exact monopole throughout. It is essential to emphasize that **one single governing equation** of energy and geometry—combining geometry and energy via T_{ij} and the pairwise sagitta—governs the entire exterior ($r \geq R$). The $\sqrt{2}R$ boundary represents a continuous geometric transition rather than a change of law.

Let $\Theta(r) = 2 \arcsin(R/r)$ be the angle the limb subtends at radius r , and define the limb opposition parameter

$$\chi(r) \equiv \cos \Theta(r) = 1 - 2 \frac{R^2}{r^2}. \quad (29)$$

Two streams from a body of radius R can arrive more than 90° apart only if $\Theta > 90^\circ$, that is, only if $r < \sqrt{2}R$ (Appendix A.5).

Table 8: The Three Regions.

Region	Monopole drift (N)	Relational sagitta (arrival geometry)	Dynamic Stress
1 ($r > \sqrt{2}R$, $\chi > 0$)	Exact $1/r^2$	Acute pairs only ($\theta_{bc} < 90^\circ$): relational momentum is strictly additive	Radial flux traded steadily into hoop stress; no opposing cancellation
2 ($R < r < \sqrt{2}R$, $\chi < 0$)	Exact $1/r^2$	Opposed pairs ($\theta_{bc} > 90^\circ$) switch on; destructive vector cancellation	Opposed-pair cancellation softens dynamic acceleration; deposited into constituent stress and dilation
3 ($r < R$)	Net flux falls smoothly to zero at the centre	Pairs extend to 180°	Cancelled flux becomes isotropic bath

The outward geometric progression and opposed pairs. Traversing outward from the stellar surface through the exterior reveals a rigorous geometric progression:

1. *Stellar surface* ($r = R$): The body subtends an entire hemisphere (2π steradians), with aperture $\Theta(R) = 2 \arcsin(1) = 180^\circ$. Arriving streams from opposite horizons arrive in direct opposition ($\theta_{bc} = 180^\circ$, $\cos \theta_{bc} = -1$), providing maximum limb opposition.
2. *Region 2* ($R < r < \sqrt{2}R$): The aperture opens past a right angle ($90^\circ < \Theta < 180^\circ$), yielding a negative limb opposition parameter $\chi(r) \equiv \cos \Theta(r) = 1 - 2R^2/r^2 < 0$. Outer chord pairs subtend obtuse angles ($\theta_{bc} > 90^\circ$, $\cos \theta_{bc} < 0$), activating destructive vector cancellation.
3. *The $\sqrt{2}R$ geometric bifurcation* ($r = \sqrt{2}R \approx 1.4142 R$): The aperture subtends exactly $\Theta = 2 \arcsin(1/\sqrt{2}) = 90^\circ$, where $\chi(\sqrt{2}R) = 0$. This defines an exact geometric threshold: beyond $\sqrt{2}R$, obtuse pair arrival ($\theta_{bc} > 90^\circ$) is geometrically impossible.
4. *Region 1* ($r > \sqrt{2}R$): The aperture is acute ($\Theta < 90^\circ$, $\chi > 0$). Every arriving pair satisfies $\theta_{bc} \leq \Theta < 90^\circ \implies \cos \theta_{bc} > 0$. Relational momentum is strictly cumulative and forward-directed, with zero destructive vector cancellation. Radial flux trades steadily into transverse hoop stress B via tensor conservation ($A' + 2(A - B)/r = 0$), but without vector opposition.
5. *Asymptotic far field* ($r \gg \sqrt{2}R$): The aperture collapses ($\Theta \approx 2R/r \rightarrow 0$), streams become nearly parallel, and the arrival tensor asymptotically approaches $T_{ij} \approx \epsilon \hat{r}_i \hat{r}_j$.

Table 9 gives the radial progression for a uniform sphere. The net drift flux $\mathbf{N}$ matches the exact monopole down to the surface; the gap between absolute flux I and net vector flux $|\mathbf{N}|$ exists at every radius (about 5% at $3R$), but the share of that gap supplied by opposed pairs switches on analytically at $\sqrt{2}R$, reaching 37% at the surface and $\frac{3}{4}$ at the centre (the value for an isotropic bath).

Table 9: Net and Absolute Flux, Analytic Limb Aperture, and Opposed-Pair Share for a Uniform Sphere (Exterior Flux Scaled by r^2 ; Analytic Aperture $\Theta(r) = 2 \arcsin(R/r)$).

r/R	3.0	2.0	$\sqrt{2} \approx 1.414$	1.38	1.2	1.0
Net flux Nr^2	1.000	1.000	1.000	1.000	1.000	1.000
Absolute flux Ir^2	1.023	1.056	1.130	1.139	1.210	1.492
Analytic aperture $\Theta(r)$	38.9°	60.0°	90.0°	92.9°	112.9°	180.0°
Opposed share of the gap	0	0	0	0.0001	0.047	0.374

The stress exchange. Equation (27) trades radial flux for hoop stress exactly. The anisotropy of the arrival tensor, $T_{rr} - T_{\perp} = \epsilon(1 - \langle r'^2 \rangle / r^2)$, peaks at $r = \sqrt{3\langle r'^2 \rangle}$, which is $1.34R$ for a uniform sphere and moves inside the body for centrally concentrated profiles. The cancelled flux is not lost; at the centre it is all isotropic stress, and this stress is what relaxation responds to.

The physics of dynamic softening. The static acceleration cannot change across the regions: it is the monopole. A body at rest receives only the net flux; the non-net part acts like a pressure from all sides and gives no push. For an extended body with bulk momentum $\mathbf{v}$ entering Region 2:

- The radial velocity components encounter a radial flux weakened by $(1 - \frac{2}{3} \frac{\langle r'^2 \rangle}{r^2})$.
- The transverse velocity components cut across the hoop stress $T_{\perp}$.
- Because opposing limb curls cancel at wide angles (approaching 180° at the surface), the net directional acceleration on the moving body is **softened**.

- By conservation of momentum flux, this cancelled directional momentum is absorbed by the entering body, manifesting as internal stress/dilation and redistribution of its constituent speed budget.

Testability. In the Solar System the regions are an ontological difference from General Relativity, not an observable one. Mercury's perihelion lies about $66 R_{\odot}$ from the Sun, far outside its Region 2, and Earth's ϵ^2 is about 10^{-19} . Compact objects are the only regime where the regions could show: for a neutron star, $\epsilon \approx 0.2$ and Region 2 runs from about 12 to 17 km, around the innermost stable orbit. There it would appear as a dependence of the near-surface exterior on internal structure, through $\langle r'^2 \rangle$, which Birkhoff's theorem forbids in General Relativity.

10 Branch B: Composition and Relaxation

Branch B adds no physics of its own. It is the rules for composing Branch A objects into macroscopic bodies, and the relaxation step by which each circulation responds to what it receives.

10.1 Three Operations with Non-Overlap

The dynamics consist of two non-overlapping operations, projection and relaxation, followed by motion:

1. **Project.** Linear, retarded projection from every source's current, relaxed state, by the Green's function (Section 5).
2. **Relax.** With the arriving field frozen, each circulation adjusts its cadence and extent locally.
3. **Move.** Each circulation drifts by stationary phase down the gradient (Eq. 6).

Retardation and adiabaticity keep projection and relaxation from ever overlapping: an atom's internal transit time ($\sim 1.5 \times 10^{-16}$ s for atomic structure, far shorter for its constituents) is vastly shorter than any change in the field, so no fixed-point equation sits inside a single step. The Green's function maps a source region to a target region that never overlaps it; within each circulation, trapped flux receives and escaping flux projects, and the border between them is the separatrix (Section 11.5).

The structure stops at second order: single streams and their pairwise geometry, unified before readout. There is one operator, applied causally to a state that already contains its own relaxation. Higher orders appear only when that state is reconstructed from unrelaxed constituents; one fold-back reconstructs it through second order, and the converged fold-back reconstructs it exactly. Thereafter all feedback is causal transit counting: with time frozen, second order suffices; with time running, each pass is the next time step.

Because projection and relaxation never overlap, the evolution is an explicit time-stepping of a retarded, volumetric field whose streams propagate at the local speed set by the accumulated ambient: from any fully specified initial state, each step is exact up to numerical precision.

10.2 Relaxation Is Additivity

A circulation responds to the additive, unified momentum flux arriving at it, the full summed tensor, never to streams one by one. The relaxation rule is therefore the additivity rule of Axiom III, applied at the receiving circulation. Respecting additivity is also what makes the cancellations of Section 4 valid.

10.3 The Universal Confinement Response

Confinement is the same property for every circulation, and the absolute level of the sea is inaccessible (Axiom III). A confined circulation must therefore respond to an increment of debt the same way, in its own local units, whatever background debt it already sits in; otherwise the background would be observable. If $f(\epsilon)$ is the factor by which a debt ϵ scales the squared cadence, this requires

$$\frac{f(\epsilon_{\text{bg}} + \delta\epsilon)}{f(\epsilon_{\text{bg}})} = f(\delta\epsilon) \quad \text{for every } \epsilon_{\text{bg}}, \quad (30)$$

whose only continuous solutions are exponentials (Appendix A.9). With the first-order coefficient fixed by the speed budget,

$$\left(\frac{d\tau}{dt}\right)^2 = e^{-2\epsilon}, \quad c_{\text{circ}}^2 = c^2 e^{-2\epsilon}, \quad dl_{\text{phys}} = e^\epsilon dl, \quad (31)$$

in the substrate chart, applied once and exactly to the unified debt. At first order the speed budget reads

$$c^2(1 - 2\epsilon(\mathbf{x})) = c_{\text{circ}}^2(\mathbf{x}) + v_{\text{trans,phys}}^2, \quad (32)$$

where $v_{\text{trans,phys}}$ is the translational speed measured with comoving rods: every unit of translational momentum gained must be surrendered by the internal cadence, $d\mathbf{p}_{\text{trans}} = -(E_{\text{circ}}/c^2) d(c_{\text{circ}}) \equiv M_0 d\mathbf{v}$, and

$$\frac{d\tau}{dt} = \frac{c_{\text{circ}}}{c} = \sqrt{1 - 2\epsilon(\mathbf{x}) - \frac{v_{\text{trans,phys}}^2}{c^2}}. \quad (33)$$

Each increment of debt taxes the circulation that remains, and the cadence never vanishes for any finite debt. The single-centre readout is

$$ds^2 = -e^{-2\epsilon} c^2 dt^2 + e^{2\epsilon} (d\bar{r}^2 + \bar{r}^2 d\Omega^2), \quad \epsilon = \frac{GM}{c^2 \bar{r}}, \quad (34)$$

with areal radius $r = \bar{r} e^\epsilon$; in the areal chart it gives $g_{00} = -(1 - 2u + \mathcal{O}(u^3))$ and $g_{rr} = 1 + 2u + \mathcal{O}(u^2)$, with $u = GM/c^2 r$. The coefficient of u^2 in g_{00} vanishes, so $\beta = 1$: for a single centre, the source's own relaxation is already contained in its measured mass, and only the shape of the response matters. A response linear in the substrate radius would give $\beta = 0$ and is excluded by Mercury. The substrate radius is $\bar{r}$ (Axiom I) and the areal radius $r = \bar{r} e^\epsilon$ is its readout; the framework commits to this pair.

Light is unconfined. The same debt acts on it without the confinement structure, which is why its deflection carries the factor that a slow body's acceleration does not (Section 11). For moving circulations, the universal confinement response couples covariantly to translational momentum through the moving speed budget (Section 12.5), preserving local proper speed c .

10.4 Relaxed Emission and the Active Mass

Projection always acts on the current state, so a relaxed circulation emits its redshifted energy together with its full momentum flux; this is the source half of the Green's function, not an extra rule. The primary physical quantity is the total momentum-flux content within a body's extent:

$$M_{\text{source}} \equiv \frac{E_{\text{total}}}{c^2} = \frac{1}{c^2} \int_V (\mathcal{E}_{\text{circ}} + \mathcal{E}_{\text{kin}} + \mathcal{E}_{\text{th}} + \mathcal{E}_{\text{bind}}) dV, \quad (35)$$

with $\mathcal{E}_{\text{bind}} < 0$. Redshift alone would remove twice the binding energy; the momentum flux of internal motion adds back three times the kinetic energy; with virial balance, binding is counted exactly once. This is the arithmetic of Tolman's mass [24] in General Relativity, obtained here from the source half of the Green's function.

10.5 The Supplied Initial Condition and the Fold-Back

Reality supplies the relaxed state. Applied to a state that already contains its own relaxation, which in practice means measured masses, one causal application of the operator is exact, and all later feedback is time evolution. Reconstructing that state from unrelaxed constituents is a separate task: project, relax, project again. For a uniform sphere, one fold-back differs from the converged state by a relative amount $(GM/c^2 R)^2$ (Table 10): negligible for the Sun, about 4% at neutron-star compactness. This is why the single-centre results below can use measured masses directly.

Table 10: One Fold-Back Compared with the Converged Self-Consistent State (Uniform Sphere, Surface Debt).

Compactness $GM/c^2 R$	2×10^{-6} (Sun)	0.01	0.1	0.2
Relative error of one fold-back $(GM/c^2 R)^2$	6×10^{-12} 4×10^{-12}	1.4×10^{-4} 1×10^{-4}	1.2×10^{-2} 1×10^{-2}	4.2×10^{-2} 4×10^{-2}

10.6 Rearrangement and Path Independence

Flux is moved, never created. In a simulation, adding an atom means bringing an existing atom in from infinity; a Cavendish experiment rearranges sources that were already contributing, and so changes gradients, never the absolute level. The steady state must not depend on the order of assembly. This holds exactly when the pair relation is symmetric, which is the same requirement as conservation.

The factor of $1/2$ appearing in incremental assembly and virial binding energy ($\mathcal{E}_{\text{bind}} = -\frac{1}{2} \sum_{b \neq c} Gm_b m_c / r_{bc}$) applies strictly to scalar state energy to avoid double-counting undirected physical bonds $\{b, c\}$. In contrast, the causal momentum ledger consists of $N(N-1)$ directed vectors ($b \rightarrow c$), where both directions of momentum transfer are physically realized, establishing Newton's third law without any halving factor.

10.7 Back-Reaction as Coarse-Graining

At constituent level no circulation acts on itself. Effective self-interaction appears only when constituents are grouped into a body and treated as one object: binding, interior relaxation, tidal response and radiation reaction then look like a body acting on itself. The precision rule is that an effective term is legitimate if and only if it maps back onto pairs of distinct constituents, each pair counted once. With no points, there is no divergent self-force to regularize; radiation reaction is the emission recoil debited at projection. Lunar laser ranging's bound on the Nordtvedt parameter (about 10^{-4}) is the direct check of this bookkeeping.

10.8 The Summed Green's Function and the Momentum-Flux Balance Law

Summed over the constituents of a body, including their internal motions, the moving kernel should reproduce the retarded balance law sourced by the full momentum-flux (stress-energy) tensor of the confined flux,

$$\square D^{\mu\nu} = -\frac{4\pi G}{c^4} T^{\mu\nu}, \quad D^{\mu\nu}(t, \mathbf{x}) = \frac{G}{c^4} \int \frac{T^{\mu\nu}(t - |\mathbf{x} - \mathbf{x}'|/c, \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} d^3 x'; \quad (36)$$

if it does, Eq. (36) is a theorem rather than a postulate: summing over constituent cycles, the stress part of the source integrates directly into the divergence-free balance law on the flat

Minkowski substrate. For a static source, $T^{00} = \rho c^2$ gives $D^{00} = U/c^2 = \epsilon$; a moving source adds $D^{0i} = V^i/c^3$, with $V^i = G \int \rho v^i / |\mathbf{x} - \mathbf{x}'| d^3x'$; internal stresses add D^{ij} . The law is Lorentz covariant.

First-order readout. With readouts built from $D_{\mu\nu}$ and $\eta_{\mu\nu}$ alone, $h_{\mu\nu} = a D_{\mu\nu} + b \eta_{\mu\nu} D$; the static readout ($\gamma = 1$, Section 10.9) fixes $b = -2$ and $a = 4$:

$$h_{\mu\nu} = 4(D_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}D), \quad (37)$$

with $h_{00} = 2U/c^2$, $h_{ij} = 2U\delta_{ij}/c^2$ and $h_{0i} = -4V_i/c^3$, identical to linearized General Relativity in harmonic gauge [9, 19]. Three consequences follow:

- *No preferred frame.* Kernel and readout are covariant, so local physics does not depend on the velocity of the system relative to the zero-momentum frame of whatever distribution has been unified: $\alpha_1 = \alpha_2 = 0$ at linear order.
- *Frame dragging.* The momentum part h_{0i} gives the gravitomagnetic field and, for a rotating body, the coefficient $h_{0\phi}$ of Section 1.4.
- *Radiation.* Conservation of the source, $\partial_\mu T^{\mu\nu} = 0$, gives $\partial_\mu D^{\mu\nu} = 0$. Of the propagating debt, only the transverse tensor part acts on instruments: there is no scalar (breathing) mode, and conservation removes monopole and dipole radiation at leading order, so the leading waveform is quadrupolar.

Interaction stresses. For a bound binary, the quadrupole formula depends on interaction stresses as well as on $\int \rho v v$. Here the flux exchanged within a bound system completes transits within it, so it is confined and sources debt (Axiom II); this internal binding flux provides the interaction stresses required by the quadrupole formula, reproducing the radiative balance tested to 10^{-4} in the double pulsar [47].

10.9 The First-Order Single-Source Geometry (Kerr–Schild Form)

For a single static source, the kernel of Eq. (10) is $h_{\mu\nu} = 2\epsilon(r)\ell_\mu\ell_\nu$. The same form is selected by the relational coupling menu, independently of the kernel, in four steps.

Step 1: Null outward flux. Outward disturbances travel radially at $c = 1$, with tangent $\ell^\mu = (1, \hat{\mathbf{r}})$ and covector

$$\ell_\mu = -\hat{t}_\mu + \hat{r}_\mu, \quad \ell^\mu\ell_\mu = 0. \quad (38)$$

Step 2: The null-dyad form. The only vectors available to a relational coupling are matter-defined: the source 4-velocity u^μ and the chord null vector ℓ^μ ; the only further tensor is $\eta_{\mu\nu}$. The most general symmetric coupling is

$$h_{\mu\nu} = a\ell_\mu\ell_\nu + b u_\mu u_\nu + \kappa(u_\mu\ell_\nu + \ell_\mu u_\nu) + d\eta_{\mu\nu}. \quad (39)$$

1. *Areal readout.* Expressing the single-source geometry with sphere areas $4\pi r^2$, and since ℓ and u have no angular components, $h_{\theta\theta} = dr^2$ must vanish: $d = 0$.
2. *No self-interaction of a stream.* A source's outward flux does not act on itself (Axiom III): $h_{\mu\nu}\ell^\nu = 0$. With $u \cdot \ell = -1$ this forces $b = 0$ and $d = \kappa$, hence $\kappa = 0$. The coupling is a pure null dyad, nilpotent for a single source; this is the Kerr–Schild condition, given a physical origin.
3. *Speed budget.* The amplitude is $a = 2\epsilon(r)$ (Step 4).

Step 3: Amplitude. By Eq. (24), $\epsilon = GM/c^2\bar{r} \approx GM/c^2r$, with $M = E_{\text{act}}/c^2$.

Step 4: Derivation of Coefficient $C = 2$. The coefficient $C = 2$ is derived directly from the speed budget and operational clock cadence. Immersed in debt ϵ , the work done by the gradient transfers kinetic capacity $\Delta(v^2) = 2c^2\epsilon$. By the invariant speed budget $c^2 = c_{\text{circ}}^2 + v^2$, this kinetic gain taxes the internal circulation by $\Delta(c_{\text{circ}}^2) = -2c^2\epsilon$, fixing the operational cadence $(d\tau/dt)^2 = c_{\text{circ}}^2/c^2 = 1 - 2\epsilon$. Because operational clocks directly measure this round-trip transit cadence, the temporal metric component is identically $g_{00} = -(d\tau/dt)^2 = -(1 - 2\epsilon)$, establishing $C = 2$ as a derived kinematic theorem of rod-clock duality rather than an empirical calibration.

Scalar gravity is excluded. Nordström's theory [13] coupled a scalar field to the trace $T^\mu{}_\mu$, giving a conformally flat geometry in which light does not deflect. Here the coupling is a null dyad of momentum flux, which gives the full 1.75''.

10.10 Synchronization and the Single-Centre Readout

With $h_{\mu\nu} = 2\epsilon\ell_\mu\ell_\nu$ and $\ell_\mu dx^\mu = -c dT + dr$, the Kerr–Schild form is linear and its inverse terminates, $g^{\mu\nu} = \eta^{\mu\nu} - 2\epsilon\ell^\mu\ell^\nu$. The outward line element is

$$ds^2 = -(1 - 2\epsilon)c^2 dT^2 - 4\epsilon c dT dr + (1 + 2\epsilon)dr^2 + r^2 d\Omega^2, \quad (40)$$

whose cross-term is the outgoing Eddington–Finkelstein term of retarded propagation. Synchronizing clocks along two-way round trips, $c dT = c dt - f'(r) dr$, removes the cross-term for

$$f'(r) = \frac{2\epsilon(r)}{1 - 2\epsilon(r)}, \quad (41)$$

and the spatial coefficient becomes

$$g_{rr} = \frac{4\epsilon^2}{1 - 2\epsilon} + (1 + 2\epsilon) = \frac{1}{1 - 2\epsilon(r)}, \quad (42)$$

giving the areal line element

$$ds^2 = -(1 - 2\epsilon(r))c^2 dt^2 + (1 - 2\epsilon(r))^{-1}dr^2 + r^2 d\Omega^2, \quad \epsilon = \frac{GM}{c^2 r}. \quad (43)$$

This is a first-order consistency check. Its exact form, in which the background radius equals the areal radius, is General Relativity's completion, not this framework's: here the substrate radius is $\bar{r}$ and the readout is Eq. (34), which coincides with Eq. (43) through first post-Newtonian order, including $\beta = 1$, and differs in the strong field (Section 13.4). In the single-centre sector the readout also coincides with Whitehead's at first post-Newtonian order [15]; the frameworks differ in what superposes (Section 14).

10.11 Momentum Conservation

Local conservation of momentum flux is an axiom. In flat-space field theories, matter's momentum flux alone is not conserved when bodies interact, and the Gupta–Feynman–Deser argument [7, 8, 10] concludes that the mediating flux must then act as a source; Whitehead's theory, lacking such a mechanism, violated momentum conservation [18]. In the present framework the momentum in transit is carried by propagating flux and debited and credited at projection and reception, and second-order structure comes from relaxation applied causally, not from the field sourcing itself. The dynamics therefore consist of two first-order operations, linear causal projection and local adiabatic relaxation; all post-Newtonian structure is their iteration in time, realized in practice through the explicit pairwise momentum-flux ledger (Section 16).

One ledger needs particular care: for an unbound encounter, such as light deflected by the Sun, the photon has an open null topology with no closed transits and sources no debt, so the Sun receives no reciprocal projection. The photon does not exchange momentum with the Sun through contact scattering; rather, it traverses the stationary-phase streamline determined by the Sun's pre-existing outward momentum flux gradient. The local divergence of total momentum flux ($\partial_\mu \Pi_{\text{total}}^{\mu\nu} = 0$) is preserved at every point along the beam as the wavevector refracts through the streaming background, without requiring a retroactive mechanical recoil on the emitting body.

11 The Matter Interface

The kinematics needs a small set of class properties of matter, not specific particle topologies. Any structure that has them falls identically, which the data require: ordinary matter is a mixture of very different internal structures, yet all of it falls alike to about 10^{-15} [20].

11.1 Six Class Properties

1. **Trapped flux.** A trapped family of flux lines, closed or not: it gives rest energy, inertia and a reception region, and separates matter from radiation.
2. **Persistence.** The structure stays bounded on its transit timescale.
3. **Balanced projection.** No net momentum is projected in the rest frame, so nothing self-propels.
4. **Net circulation.** A definite angular momentum (spin), which feeds the gravitomagnetic part and averages to zero in symmetric ensembles.
5. **Universal confinement response.** The same response to debt in its own units, whatever the background (Section 10.3).
6. **Isochrony.** Commensurate transit periods across the continuum of transit paths within the structure, giving one rest frequency, $\hbar\omega_0 = m_0c^2$.

Which structure each particle has, its mass and its lifetime are outside the kinematic scope; masses enter as measured.

11.2 Persistence Gives Weight Equal to Inertia

Local conservation and boundedness give the virial identity (Appendix A.8)

$$\int T^{ij} dV = \frac{1}{2} \frac{d^2}{dt^2} \int T^{00} x^i x^j dV. \quad (44)$$

For a bounded structure the right-hand side averages to zero over a cycle, so the integrated momentum flux vanishes: von Laue's condition [23]. By Laue's theorem, a stationary, self-confined system's total energy and momentum then transform as those of a point particle of mass E/c^2 ; cycle-averaged, its coupling to a slowly varying debt depends on its total confined energy alone, and its weight equals its inertia. No field model or stability theorem is needed. Free radiation has positive integrated momentum flux, equal to its energy, with nothing to balance it; that is why light responds to the same debt with twice the deflection of a slow body.

11.3 A Worked Transit Loop

Take flux running between two turning points a distance L apart along the gradient. Flux carries momentum nE/c , with local index $n \approx 1 + 2\epsilon$, so at the near turning point, where the debt is deeper, it carries more momentum per unit of conserved energy than at the far point. Turning it back delivers $2n_{\text{near}}E/c$ toward the source at one end and $2n_{\text{far}}E/c$ away from it at the other; over a round trip of duration $2L/c$ the loop length cancels, and the flux alone receives $2E|\nabla\epsilon|$ toward the source, twice the matter value, which is the response of free light. In a persistent structure the confining stress, whose integral cancels the flux's own momentum flux (Eq. 44), receives the spatial share with opposite sign, $-E|\nabla\epsilon|$. The net is $E|\nabla\epsilon|$ and, with inertia E/c^2 , the drift is $\mathbf{a} = c^2\nabla\epsilon$. The drift is delivered at the turning points, which links internal motion to bulk motion, and the factor of two between free and confined flux comes from the same rule.

11.4 Particles Carry the Flux; Atoms Organise It

Elementary constituents' speed- c , Compton-scale circulations carry almost all of a body's energy and produce the leading sagitta. Atoms make the curl symmetric and keep it located; their churning, at speeds of order αc , carries a fraction of order α^2 of the energy and sets the binding fraction. Transit ratios link the levels: size ratio equals speed ratio times transit-time ratio, so for hydrogen $\alpha \times \alpha^{-2} = \alpha^{-1}$, the Bohr radius over the reduced Compton wavelength. Field-of-view corrections therefore scale with the Compton size and are utterly negligible; the observable geometry lives at body scale.

11.5 The Separatrix and the Control Surface

Each persistent structure's flux lines divide into a trapped family, which receives, and an escaping family, which projects, separated by a separatrix defined in the structure's own rest frame. This is non-overlap as a property of topology, with no preferred frame. Reception is evaluated across a control surface drawn just outside the separatrix: ambient streams cross it freely, projected flux leaves through it, and the structure's own confining stress, which integrates to zero by Eq. (44), lies inside it. The drift is the net momentum flux through that surface.

11.6 Hydrostatic Balance

Self-gravitating equilibrium follows without presupposing G : in response to the collective drift, non-superimposing constituents establish an internal kinetic momentum flux $\mathbf{\Pi} = \rho\mathbf{v} \otimes \mathbf{v} + P_{\text{th}}\mathbb{I}$, and virial balance requires

$$\nabla \cdot \mathbf{\Pi} + \rho \mathbf{a}_{\text{total}} = \mathbf{0} \iff \frac{dP_{\text{th}}}{dr'} = -\rho(r')|\mathbf{a}_{\text{total}}(r')|, \quad (45)$$

the Newtonian hydrostatic limit, valid for $P/\rho c^2 \ll 1$. Ordinary stars are held up by matter's own pressure, as in General Relativity.

12 Rod-Clock Kinematics, the Interior, and the Light Cone

12.1 Operational Definitions

All measurements are matter-on-matter interactions:

- **Operational clocks.** A physical clock is the round-trip phase transit time $\Delta\tau = \oint dl/c$ of speed- c momentum flux circulating within a bound state; transition frequencies scale with constituent mass-energy, $\omega_{\text{clock}} \propto m_0 c^2 / \hbar$.

- **Operational measuring rods.** A physical rod is the spatial separation between atoms in a lattice, set by the extent of electronic circulation.
- **Charts.** The substrate chart is the causal arena on which the debt spreads (Axiom I). The areal chart, with radial coordinate r defined by rod-measured sphere areas $4\pi r^2$, is derived; for a single centre, $r = \bar{r}(1 + \epsilon)$ at first order.

12.2 Debt, Local Streaming, and Instrument Verification

The derivations here are in the areal chart at first post-Newtonian order. By the direct integral, the debt satisfies

$$\epsilon(r) = \frac{1}{c^2} \int_r^\infty |\mathbf{a}_0(r')| dr' \quad (46)$$

to first order, $\epsilon = GM/c^2 r$ in the exterior and continuing smoothly to the core inside matter. The local radial streaming through radius r is

$$\zeta(r) \equiv \frac{r|\mathbf{a}_0(r)|}{c^2} = \frac{Gm(r)}{c^2 r}. \quad (47)$$

In the exterior $\zeta \equiv \epsilon$ at first order; inside matter, because $m(r')$ is non-decreasing,

$$\epsilon(r) = \frac{1}{c^2} \int_r^\infty \frac{Gm(r')}{r'^2} dr' > \zeta(r) \quad (r < R). \quad (48)$$

Clock dilation and transverse speed. Immersed in a debt, a circulation's coordinate cadence is taxed, $d\tau = \sqrt{1 - 2\epsilon} dt$, while its rest mass is preserved (persistence). Along transverse directions in the areal chart rods are uncontracted, so preserving local proper speed c fixes the transverse coordinate light speed

$$v_\perp = c \frac{d\tau}{dt} = c\sqrt{1 - 2\epsilon(r)}. \quad (49)$$

Radial rod contraction. A comoving rod calibrated by two-way transit, $\Delta r_{\text{phys}} = \frac{1}{2}c \Delta\tau$, with exterior radial light speed $v_r = c(1 - 2\epsilon)$, gives

$$\Delta r_{\text{phys}} = \frac{1}{2}c \left(\frac{2\Delta r}{v_r} \right) \sqrt{1 - 2\epsilon} = \frac{\Delta r}{\sqrt{1 - 2\epsilon(r)}}, \quad (50)$$

matching $\sqrt{g_{rr}}\Delta r$. Physically, rod contraction represents the standing-wave compression of electronic and lattice circulations under the directional speed budget, contracting in step with clock dilation to preserve local proper light speed c .

12.3 The Areal Compensation and the Interior

Inside an extended body, the mass decomposes into outer shells and an enclosed core.

Outer shells. For a point inside a shell of mass m_s and radius a , the shell's streams arrive from all directions. Using the interior shell average $\langle L \rangle = a + r^2/3a$, Eq. (28) gives $T_{ij} = \frac{1}{3}\epsilon_{\text{shell}} \delta_{ij}$: isotropic and constant throughout the shell's interior. An isotropic, constant debt is a uniform change of units, so outer shells tax clocks through ϵ but leave the interior's relative spatial read-out unchanged.

The areal compensation. Transverse rods respond to the transverse part of the arrival tensor and define the areal radius, $r_A = r(1 + T_\perp)$. Converting radial lengths to the areal chart gives

$$g_{rr} - 1 = 2[T_{rr} - T_\perp - r T'_\perp]. \quad (51)$$

With the exterior arrival tensor (Eq. 26), the extended-body terms cancel exactly and $g_{rr} = 1 + 2\epsilon$ down to the surface. Inside matter the same identity gives

$$g_{rr} = 1 + 2\zeta + \mathcal{O}(\epsilon^2), \quad (52)$$

continuous through the surface; Table 11 shows the check for two profiles. This coincides with General Relativity's areal interior metric at first order. Its exact form follows from the response: the exponential gives $g_{rr} = (1 - \zeta)^{-2}$, where General Relativity gives $(1 - 2\zeta)^{-1}$. The areal chart is regular wherever $\zeta < 1$; inside the Sun, $\zeta \lesssim 4.2 \times 10^{-6}$.

Table 11: Interior and Exterior $g_{rr} - 1$ from the Areal Compensation, Compared with $2m(r)/r$ (Units $GM/c^2 R$, r in Body Radii).

r/R	Uniform		Centrally concentrated	
	$g_{rr} - 1$	$2m/r$	$g_{rr} - 1$	$2m/r$
0.2	0.0800	0.0800	0.4883	0.4883
0.8	1.2800	1.2800	2.4129	2.4129
1.5	1.3333	1.3333	1.3333	1.3333

Surface and origin. Interior and exterior readouts are readouts of a single integral, so g_{00} and g_{rr} are continuous without junction conditions. With finite central density, $\zeta \propto \bar{r}^2 \rightarrow 0$, so $g_{rr}(0) = 1$ and

$$\lim_{r \rightarrow 0} \frac{2\pi r}{\int_0^r \sqrt{g_{rr}} dr'} = 2\pi, \quad (53)$$

with no conical defect at the origin.

12.4 Loop Orientation Invariance

An operational clock must tick at the same rate whatever its orientation. In the substrate chart this is immediate. In the areal chart, for a loop element of physical length dl_{phys} at angle θ to the radial line, $dr = dl_{\text{phys}} \cos \theta \sqrt{1 - 2\zeta}$ with $v_r = c\sqrt{(1 - 2\epsilon)(1 - 2\zeta)}$, and $r d\phi = dl_{\text{phys}} \sin \theta$ with $v_\perp = c\sqrt{1 - 2\epsilon}$:

$$c^2 dt^2 = \frac{dr^2}{(1 - 2\epsilon)(1 - 2\zeta)} + \frac{(r d\phi)^2}{1 - 2\epsilon} = \frac{dl_{\text{phys}}^2}{1 - 2\epsilon} (\cos^2 \theta + \sin^2 \theta), \quad (54)$$

so $dt/dl_{\text{phys}} = 1/(c\sqrt{1 - 2\epsilon})$ for every orientation and tumbling state.

12.5 The Moving Loop and the Speed Budget

For a loop translating with $\mathbf{v}_{\text{trans}} = (v_r, \mathbf{v}_\perp)$, the budget of Eq. (32) is isotropic in physical velocities; converting with radial rod contraction gives $v_{\text{trans,phys}}^2 = v_r^2/(1 - 2\zeta) + v_\perp^2$. Internal flux is bounded by the local null cone,

$$\frac{u_r^2}{c^2(1 - 2\epsilon)(1 - 2\zeta)} + \frac{|\mathbf{u}_\perp|^2}{c^2(1 - 2\epsilon)} = 1. \quad (55)$$

With $\mathbf{u} = \mathbf{v}_{\text{trans}} + \mathbf{c}_{\text{circ}}$ averaged over a cycle, the linear cross-terms vanish by the odd parity of closed sweeps, and by Section 12.4

$$\left(\frac{d\tau}{dt}\right)^2 + \frac{v_r^2}{c^2(1-2\zeta(r))} + \frac{|\mathbf{v}_\perp|^2}{c^2} = 1 - 2\epsilon(r). \quad (56)$$

Multiplying by $-c^2 d\tau^2$,

$$-c^2 d\tau^2 = -(1-2\epsilon(r))c^2 dt^2 + (1-2\zeta(r))^{-1} dr^2 + r^2 d\Omega^2 \equiv \tilde{g}_{\mu\nu} dx^\mu dx^\nu, \quad (57)$$

which reduces to Eq. (43) in the exterior. By stationary phase, the translating loop extremizes proper time.

12.6 Wave Mechanics: The Eikonal Light Cone

High-frequency waves obey $\tilde{g}^{\mu\nu} \partial_\mu S \partial_\nu S = 0$. In the areal chart at first order, radial rays move at $v_r = c\sqrt{(1-2\epsilon)(1-2\zeta)}$ (reducing to $c(1-2\epsilon)$ in the exterior) and transverse rays at $v_\perp = c\sqrt{1-2\epsilon}$ (Theorem A.5). Wave mechanics and matter kinematics share one light cone.

13 The Classical Tests

The tests of Sections 13.1–13.3 and 13.5–13.6 are first-post-Newtonian results. Section 13.4 states the strong-field consequences of the exponential response as conjectures.

13.1 Perihelion Precession of Mercury

13.1.1 The velocity-dependent scalar-potential barrier

In classical flat-space mechanics, a velocity-dependent scalar potential $V(r, \dot{\phi}) = -GM/r - GML^2/(c^2 r^3)$ yields canonical momentum $p_\phi = mr^2 \dot{\phi} (1 + 2GM/c^2 r)$ and, through the coordinate-time Binet transformation, a secular advance

$$\Delta\phi = \frac{2\pi GM}{c^2 p} \quad (\mathcal{C} = 1 \implies 14.3''/\text{century}). \quad (58)$$

The limitation comes from the form of the potential, not from point-ness. Section 13.1.3 shows that the temporal component $g_{00} = -(1-2\epsilon)$ on a rigid spatial grid produces $\mathcal{C}_{\text{areal}}^{\text{time}} = 2$ (28.7''/century), and that radial rod contraction adds $\mathcal{C}_{\text{areal}}^{\text{space}} = 1$ (14.3''/century), recovering $\mathcal{C} = 3$.

13.1.2 Relaxation of an orbiting assembly

Mercury is an extended assembly of circulations on an eccentric orbit ($e = 0.2056$) between perihelion ($r_p \approx 4.60 \times 10^{10}$ m) and aphelion ($r_a \approx 6.98 \times 10^{10}$ m), along which $\epsilon = GM/c^2 r$ varies continuously. The speed budget (Eq. 56) redistributes between internal cadence and orbital velocity, and the assembly's circulations relax adiabatically to the anisotropic readout: radial rods contract as $dr_{\text{phys}} = dr/\sqrt{1-2\epsilon}$ while transverse rods are uncontracted. In terms of proper time, the canonical momentum conjugate to ϕ on the areal chart is unshifted,

$$p_\phi = mr^2 \frac{d\phi}{d\tau} \equiv mL = \text{const.} \quad (59)$$

With conserved energy $E = c^2(1 - 2\epsilon) dt/d\tau$ and $\tilde{g}_{\mu\nu}u^\mu u^\nu = -c^2$ in the exterior,

$$-\frac{E^2}{c^2(1 - 2\epsilon)} + (1 - 2\epsilon)^{-1} \left(\frac{dr}{d\tau} \right)^2 + \frac{L^2}{r^2} = -c^2, \quad (60)$$

and multiplying by $g^{rr} = 1 - 2\epsilon$,

$$\left(\frac{dr}{d\tau} \right)^2 = \left(\frac{E^2 - c^4}{c^2} \right) + 2\epsilon c^2 - \frac{L^2}{r^2} + \mathbf{2\epsilon \frac{L^2}{r^2}}. \quad (61)$$

The bold term comes from the radial rod readout across the debt gradient. With $u = 1/r$ and $dr/d\tau = -L du/d\phi$, differentiating gives, to first post-Newtonian order,

$$\frac{d^2u}{d\phi^2} + u = \frac{GM}{L^2} + \frac{3GM}{c^2} u^2. \quad (62)$$

Mathematically this is the geodesic of the effective metric for a test body; the relaxation of the assembly under its own bulk momentum is its physical content. The $3GMu^2/c^2$ advance is entirely an effect of Mercury's translational momentum crossing the scalar monopole gradient, and is completely decoupled from the Sun's subtended visual aperture $(R_\odot/r)^2$, which is four orders of magnitude smaller and non-resonant (Section 8).

13.1.3 Chart dependence of the precession partition

In isotropic PPN coordinates, $\mathcal{C} = (2 + 2\gamma - \beta)/3 = 1$ partitions as

$$\mathcal{C}_{\text{iso}} = 1_{\text{time}} (\beta = 1) + 2_{\text{space}} (\gamma = 1) = 3 \iff 43.0''/\text{century}. \quad (63)$$

In the areal chart, suppressing radial rod contraction ($g_{rr} = 1$) while keeping $g_{00} = -(1 - 2\epsilon)$ leaves

$$\left(\frac{dr}{d\tau} \right)^2 \approx \left(\frac{E^2 - c^4}{c^2} \right) + 2GMu - \left(L^2 - \frac{4G^2M^2}{c^2} \right) u^2, \quad (64)$$

an advance of $4\pi GM/c^2p$ ($\mathcal{C}_{\text{areal}}^{\text{time}} = 2$); restoring radial contraction adds the cubic term and $\mathcal{C}_{\text{areal}}^{\text{space}} = 1$. The total is chart-independent; the attribution is not:

$$\mathcal{C}_{\text{total}} = \underbrace{1_{\text{time}} + 2_{\text{space}}}_{\text{isotropic chart}} = \underbrace{2_{\text{time}} + 1_{\text{space}}}_{\text{areal chart}} = 3. \quad (65)$$

13.1.4 Perturbative solution of the Binet equation

With $u = u_0 + \delta u$, $u_0 = (1 + e \cos \phi)/p$ and $p = a(1 - e^2)$,

$$\frac{d^2(\delta u)}{d\phi^2} + \delta u = \frac{3GM}{c^2p^2} (1 + 2e \cos \phi + e^2 \cos^2 \phi). \quad (66)$$

The resonant term gives $\delta u = (3GMe/c^2p^2) \phi \sin \phi$, so

$$u(\phi) \approx \frac{1}{p} \left[1 + e \cos \left(\phi \left(1 - \frac{3GM}{c^2p} \right) \right) \right], \quad \Delta\phi = \frac{6\pi GM}{c^2a(1 - e^2)} \text{ per orbit}, \quad (67)$$

43.0''/century for Mercury [2].

13.2 Achromatic Light Deflection

In the areal chart starlight traverses the light cone

$$c^2 dt^2 = \frac{dr^2}{(1-2\epsilon)^2} + \frac{r^2 d\phi^2}{1-2\epsilon}. \quad (68)$$

Along $y = b$ with $r = \sqrt{x^2 + b^2}$,

$$c dt \approx \left[1 + \epsilon(r) + \epsilon(r) \frac{x^2}{r^2} \right] dx \implies n(x, b) \approx 1 + \epsilon(r) \left(1 + \frac{x^2}{r^2} \right), \quad (69)$$

and by Fermat's principle

$$\theta = \left| \int_{-\infty}^{\infty} \frac{\partial n}{\partial b} dx \right| = \frac{GM}{c^2} \int_{-\infty}^{\infty} \left[\frac{b}{(x^2 + b^2)^{3/2}} + \frac{3bx^2}{(x^2 + b^2)^{5/2}} \right] dx = \frac{4GM}{c^2 b}. \quad (70)$$

Half comes from clock dilation and half from radial rod contraction; in the substrate chart the same result follows from the isotropic index $n \approx 1 + 2\epsilon$. At $b = R_\odot$, $\theta \approx 1.75''$. The deflection is achromatic.

13.3 Radar Shapiro Delay

Along the grazing path from $-r_1$ to r_2 ,

$$c\Delta t - (r_1 + r_2) = \frac{GM}{c^2} \int_{-r_1}^{r_2} \left[\frac{1}{\sqrt{x^2 + b^2}} + \frac{x^2}{(x^2 + b^2)^{3/2}} \right] dx, \quad (71)$$

with antiderivative $2 \ln(x + \sqrt{x^2 + b^2}) - x/\sqrt{x^2 + b^2}$. For $r_1, r_2 \gg b$, the round-trip delay is

$$\Delta t_{\text{Shapiro}} = \frac{4GM_\odot}{c^3} \left[\ln \left(\frac{4r_1 r_2}{b^2} \right) - 1 \right], \quad (72)$$

matching the Cassini measurement $\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$ [52]; the offset -1 is the areal-chart value to $\mathcal{O}(\epsilon)$.

13.4 Strong Fields: Photon Sphere, Shadow, and Horizon (Conjectures)

The substrate is flat, so no geometric singularity can arise. The strong-field exterior follows from the exponential response; its consequences are stated as conjectures, because the covariant response and the explicit many-body checks are not yet complete. For a static, spherically symmetric readout with $A \equiv -g_{00}$, circular null orbits satisfy $rA'(r) = 2A(r)$ and $b_{\text{crit}} = r_{\text{ph}}/\sqrt{A(r_{\text{ph}})}$. For Eq. (34), $b(\bar{r}) = \bar{r} e^{2GM/c^2 \bar{r}}$, minimized at $\bar{r} = 2GM/c^2$:

$$r_{\text{ph}} = 2\sqrt{e} \frac{GM}{c^2} \approx 3.30 \frac{GM}{c^2}, \quad b_{\text{crit}} = 2e \frac{GM}{c^2} \approx 5.44 \frac{GM}{c^2}, \quad (73)$$

about 4.6% larger than Schwarzschild's $3\sqrt{3}GM/c^2$. Clocks near the most compact objects slow but never stop, so light emitted there reaches distant observers with finite redshift, and no horizon forms. The vacuum exponential readout has an areal minimum at $\bar{r} = GM/c^2$, a throat at $r = eGM/c^2$ [32]. Current shadow measurements of M87* and Sgr A* [53, 54] agree with the Schwarzschild value at roughly the 10% level; inspiral and ringdown waveforms, the absence of echoes, and the absence of thermal surface emission from Sgr A* [49, 50] also bear on horizonless objects, as does the question of what supports objects of 10^6 to 10^{10} solar masses. What the objects imaged at horizon scale are in this framework is open, and the framework does not yet claim to pass or fail these tests.

13.5 Terrestrial Manifestation: The Geoid

A clock co-rotating with Earth's surface at $\mathbf{v}_\perp = \boldsymbol{\omega}_{\text{rot}} \times \mathbf{r}$ has cadence, by Eq. (56),

$$\left(\frac{d\tau}{dt}\right)^2 \approx 1 - 2\epsilon(\mathbf{r}) - \frac{|\mathbf{v}_\perp|^2}{c^2} = 1 + \frac{2\Phi_{\text{eff}}}{c^2}, \quad \Phi_{\text{eff}} \equiv \Phi_{\text{grav}} - \frac{1}{2}|\mathbf{v}_\perp|^2. \quad (74)$$

Earth's fluid surface relaxes until Φ_{eff} is constant along it, so clock cadence is uniform across the geoid to first order [51], from hydrostatic equilibrium on flat spacetime.

13.6 Geodetic Precession and Frame Dragging

A gyroscope is a persistent circulation whose axis is preserved relative to local rods and clocks, so its spin is Fermi-Walker transported in the effective readout. At first post-Newtonian order this gives the geodetic precession

$$\boldsymbol{\Omega} = \frac{3}{2} \frac{GM}{c^2 r^3} (\mathbf{r} \times \mathbf{v}), \quad (75)$$

about $6.6''$ per year for Gravity Probe B, as measured [56]. The momentum part of the kernel ($h_{0i} = -4V_i/c^3$) gives frame dragging with the coefficient of linearized General Relativity, also measured by Gravity Probe B. Deriving Fermi-Walker transport directly from circulation kinematics, with each circulation's action held fixed during relaxation, is future work.

14 PPN Parameters, Binding, and Multi-Source Readout

14.1 PPN Parameters

The readout gives $\gamma = 1$ (Section 10.9) and $\beta = 1$ (Section 10.3), so $(2 + 2\gamma - \beta)/3 = 1$ and the precession is $43.0''/\text{century}$. For moving sources the first-order readout gives $g_{0i} = -4V_i/c^3$ in harmonic gauge, equivalently $-\frac{7}{2}V_i - \frac{1}{2}W_i$ in standard PPN gauge, so with $\gamma = 1$, $\alpha_1 = \alpha_2 = 0$ at linear order. The Whitehead parameter $\xi = 0$ is an architectural consequence of pre-readout unification: because flux superposes linearly into the ambient sea prior to instrument readout, uniform backgrounds simply rescale base units without leaving residual directional dyad shears. The Nordtvedt parameter $\eta = 4\beta - \gamma - 3 = 0$ follows from relaxed emission of binding flux (Section 10.4). Multi-body interactions close via pairwise triad chains ($N(N-1)$ directed channels), which guarantees exact bilateral conservation ($\mathbf{F}_{ab} = -\mathbf{F}_{ba}$) and recovers the Einstein-Infeld-Hoffmann equations of motion.

14.2 Free Fall, Active Mass, and the Nordtvedt Parameter

Universality of free fall follows from the universal confinement response with persistence and conservation (Section 11) and from stationary phase, in which m_0 drops out; MICROSCOPE tests it to about 10^{-15} [20]. The active-mass sector is tested by Kreuzer-type experiments [21] and by lunar laser ranging of the Moon's composition [22]; by local conservation, the ratio of active to passive mass is universal, as these require to about 10^{-12} . The Nordtvedt parameter $\eta = 4\beta - \gamma - 3$ [19], bounded by lunar laser ranging at about 10^{-4} , vanishes identically ($\eta = 0$) due to $\beta = \gamma = 1$ and relaxed emission, which accounts for binding energy in active mass.

14.3 What Superposes: Relational Readout versus Metric Superposition

Whitehead [14] built a metric from the linear sum of retarded null dyads,

$$g_{\mu\nu}^W = \eta_{\mu\nu} + \sum_a \frac{2Gm_a}{c^2 r_a} \ell_\mu^{(a)} \ell_\nu^{(a)}. \quad (76)$$

Eddington [15] showed that for an isolated star this is Schwarzschild in Eddington–Finkelstein coordinates; Will [17] showed that for many bodies it fails, because distant masses introduce a directional term into local measurements ($\xi = 1$), predicting anomalous Earth tides [18]. The present framework avoids this failure through its core ontology:

1. **Effective metrics are readouts.** They have no causal agency and are never added (Axiom III).
2. **The distribution is unified, then read out once.** Debts add on the substrate; each circulation responds to the unified, additive flux.
3. **Backgrounds are a change of units and truncation margins define isolated systems.** In physical reality, there is always more flux than can be accounted for; one must define an isolated distribution with appropriate truncation margins. Because momentum flux is linearly superposed into an ambient sea *prior* to rod-clock readout, uniform external backgrounds simply rescale base units without leaving residual directional dyad shears, identically yielding $\xi = 0$.

14.4 Instrument Readout in Multi-Source Fields

Sources superpose linearly into one unified debt, $\epsilon(\mathbf{x}) = \sum_k \epsilon_k(\mathbf{x})$ for static sources. At first order:

1. **Clocks:** $d\tau/dt = \sqrt{1 - 2\epsilon}$.
2. **Rods:** respond to the unified arrival tensor; by Eq. (28) this is $dl_{\text{phys}} = (1 + \epsilon) dl$ equally in every direction, together with the substrate-readout displacement $\nabla\Xi$, which no instrument sees at this order.
3. **Light:** $n \approx 1 + 2 \sum_k \epsilon_k$, so deflections add source by source, each giving $4Gm_k/c^2 b_k$; this covers limb grazing and binary microlensing.
4. **Backgrounds:** a uniform contribution is a change of units.

14.4.1 Relational Free Fall

When a system falls freely within a larger environment, the environment enters only through the common free fall. A nearly uniform external debt rescales every internal clock and rod uniformly, so an environment such as the Galaxy at the Solar System produces no direction dependence in local couplings at linear order; tidal gradients remain. Because kernel and readout are covariant, uniform motion relative to the environment is invisible to internal readouts ($\alpha_1 = \alpha_2 = 0$). Light crossing an environment over scales where its debt varies is not an internal readout, so deflection by galaxies is unaffected.

15 Parsimony and Explanatory Economy

For the static single-centre tests through first post-Newtonian order and in the linearized sector, the framework makes the same predictions as General Relativity. Its claims there are not of empirical superiority but of explanatory economy:

1. **One kind of thing.** General Relativity has matter and geometry, and the energy and momentum of geometry cannot be localized. Here there is only momentum flux: matter is confined flux, the ambient sea is the flux of all matter, the debt is the state of that flux relative to the ambient level, and geometry is the readout of instruments made of flux. All energy and momentum have a definite location, and on the flat substrate local conservation integrates to exact global conservation at every scale.
2. **One substrate.** All physics takes place on flat Minkowski spacetime. Curvature is removed from the ontology, though not from what is observed.
3. **Short axioms.** Three axioms, two stated rules and one calibration (G); the exterior monopole, the interior metric, and the clock coefficient $C = 2$ are derived, and the equivalence principle follows from the response rule.
4. **Clocks, rods and inertia share one mechanism.** All follow from reallocating a single speed budget.
5. **Geodesic motion from phase.** Extremal proper time follows from stationary phase of internal circulation; attraction is drift toward slower clocks.
6. **No geometric singularity.** The substrate cannot become singular, and under the exponential response clock cadence never vanishes.
7. **One integral, inside and outside.** A single Green's function covers interior and exterior, and surface continuity is automatic.
8. **Convergence by an independent route.** General Relativity obtains the exterior Schwarzschild geometry from its field equations via Birkhoff's theorem [12]. Here the exterior monopole follows from direct integration of the Green's function, while the arrival tensor T_{ij} retains memory of internal structure ($\langle r'^2 \rangle$), breaking Birkhoff's theorem and metric blindness.
9. **A qualitative account of gravitational weakness.** Gravitation is the residual of circulating flux whose first-order components cancel, while the primary interactions engage the full flux; it is therefore far weaker. This explains the direction of the hierarchy, not its magnitude.

16 Empirical Demarcation and Domain Boundaries

16.1 Empirical Equivalence Across All Current Tests

Within the domain of current empirical verification, the momentum-flux framework is empirically equivalent to General Relativity across every tested regime. The classical and precision tests recovered from the three axioms, two stated rules, and one calibration (G) include:

- **Perihelion precession of Mercury:** The full $43.0''/\text{century}$ advance is recovered ($\mathcal{C}_{\text{areal}} = 2_{\text{time}} + 1_{\text{space}} = 3$), resulting from the adiabatic relaxation of an orbiting assembly within the unified debt gradient.
- **Achromatic deflection of starlight:** Exactly $4GM/c^2b$ ($1.75''$ at the solar limb), partitioned equally between clock cadence dilation and standing-wave radial rod contraction.
- **Radar Shapiro time delay:** The logarithmic delay matching Cassini spacecraft measurements ($\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$).
- **Gyroscopic precessions:** Geodetic precession ($6.6''/\text{year}$) and frame dragging matching Gravity Probe B data, fixed by the boost of the covariant kernel.

Table 12: The Input Ledger: Comparative Parsimony Analysis.

Category	Momentum-Flux Framework (This Work)	General Relativity
Entities	Momentum flux only	Matter (stress–energy) and geometry (metric)
Axioms	I: flat substrate, invariant c ; II: confined speed- c flux, one speed budget; III: additivity, local conservation, relative debt, unify then read out	Dynamic pseudo-Riemannian manifold; Einstein field equations; equivalence principle
Stated rules	Projection rule (Green’s function, largely fixed by additivity); response rule (universal confinement)	Clock hypothesis
Calibrations	G only (matches GR)	G
Exterior of a sphere	Static monopole derived by direct integration; arrival geometry breaks Birkhoff via $\langle r'^2 \rangle$	Birkhoff’s theorem
Second-order structure	Relaxation applied causally; singular generator and relational pairwise channels	Nonlinear field equations
Conservation	Local, for matter plus propagating flux; exact global conservation on the flat substrate	$\nabla_\mu T^{\mu\nu} = 0$; conserved totals only in special cases
Motion principle	Stationary phase of internal circulation	Geodesic motion
Universality of free fall	From the universal confinement response, with persistence and conservation	Postulated (equivalence principle)
Results in common scope	43.0''/cy; 1.75''; Shapiro delay; geoid; geodetic precession; frame dragging; $\beta = \gamma = 1$; $\xi = \eta = \alpha_1 = \alpha_2 = 0$; tensor polarizations; additive lensing; interior g_{rr} at first order	Identical
Departures	Near-limb Region 2 dynamic softening; asymptotic strong-field compression ($c_{\text{circ}} = c e^{-\epsilon}$); no horizon, photon sphere $\approx 3.30 GM/c^2$	$3 GM/c^2$; event horizon; singular collapse
Frontiers	Running-code $N(N - 1)$ pairwise simulation; observational confrontation of asymptotic compression against EHT shadow and ringdown data	—

- **Terrestrial geoid cadence:** Strict uniformity of clock rates on the hydrostatic geoid surface.
- **Universality of free fall:** Verified to 10^{-15} (MICROSCOPE), following from the universal confinement response, persistence, and stationary phase.
- **Parametrized Post-Newtonian (PPN) baseline:** The full set of linear and post-Newtonian parameters matches the empirical baseline: $\gamma = 1$, $\beta = 1$, $\alpha_1 = \alpha_2 = 0$ (no preferred frame at linear order), $\xi = 0$ (pre-readout unification), and $\eta = 0$ (relaxed emission of binding flux).

Across this entire verified domain, the framework claims explanatory economy rather than empirical conflict: it accounts for these phenomena on an uncurved flat Minkowski substrate with a single entity—momentum flux—without requiring spacetime curvature or field self-sourcing.

16.2 The Two Domains of Physical Departure

Physical departures from General Relativity are restricted to two distinct, unverified regimes:

1. Near-limb extended dynamics (Region 2 softening). For an extended body with bulk momentum traversing the near-limb exterior ($R < r < \sqrt{2}R$), the aperture opens past 90° ($\chi < 0$), activating opposing limb pairs. The destructive vector cancellation of opposing limb curls weakens the effective directional acceleration. By total momentum flux conservation, the cancelled directional momentum is deposited as internal stress and constituent dilation within the probe's circulating constituents. In the Solar System, where orbits remain far outside $\sqrt{2}R_\odot$, this effect is imperceptible; it represents a genuine physical departure observable only near compact objects (such as neutron stars) where Region 2 extends into the exterior vacuum.

2. Asymptotic strong-field compression vs. event horizons. In General Relativity, complete gravitational collapse terminates in an event horizon and geodesic incompleteness (spacetime singularities). In the present framework, the flat Minkowski substrate forbids geometric singularities, while the universal confinement speed budget is exponential:

$$c_{\text{circ}}(r) = c e^{-\epsilon(r)} > 0 \quad \text{for all finite } \epsilon. \quad (77)$$

Consequently, coordinate clock cadence and circulation speed never reach zero at any finite radius or debt. Matter undergoing extreme collapse decelerates asymptotically in cadence and undergoes **asymptotic spatial compression and continuous slowing down**. Rather than freezing at a mathematical horizon or terminating at a singularity, matter asymptotically compresses toward extreme density, with light emitted from near-surface layers escaping with finite, albeit high, redshift. Circular null orbits produce a photon sphere at $r_{\text{ph}} \approx 3.30 GM/c^2$ with critical impact parameter $b_{\text{crit}} \approx 5.44 GM/c^2$ (Section 13.4), providing an empirical demarcation against the Schwarzschild values.

16.3 Computational and Astrophysical Programme

The demarcation between this framework and curved-spacetime geometries establishes two concrete forward-looking programmes:

1. **Computational $N(N - 1)$ pairwise momentum flux simulation.** Implementing the divergence-free momentum flux ledger as running code with exact invariant checks. Because multi-body interactions close at $N(N - 1)$ directed pairs via triad chains, simulating matter plus flux in transit provides a computationally robust, self-conserving alternative

to post-Newtonian metric expansions, explicitly verifying the divergence-free balance law and binary interaction stresses.

2. **Astrophysical confrontation of horizonless compression.** Confronting the horizonless asymptotic compression phenomenology against Event Horizon Telescope (EHT) shadow observations of M87* and Sgr A*, as well as gravitational-wave accretion ringdown signatures and the absence of thermal surfaces in compact dark objects.

17 Conclusion

In conventional gravitational source modelling, symmetric curl enters only through its contribution to total energy and stress, and its geometric sagitta residual is not made explicit. Restoring it, with each circulation's sweeps treated as source-centred arcs whose sideways components cancel while their inward curvature adds, gives a Green's function whose $1/L$ kernel additivity of momentum flux independently requires. Integrated directly over an extended body, it gives the exact inverse-square monopole, deriving the shell theorem from first principles, while spherical parity and conservation forbid time-dependent monopole radiation. However, the same integral yields an exact arrival geometry: the arrival tensor T_{ij} breaks Birkhoff's theorem by retaining geometric memory of the body's interior ($\langle r'^2 \rangle$), trading radial flux for transverse hoop stress near a body.

On that basis, three short axioms on flat Minkowski spacetime, with two stated rules and one calibration (G), recover the classical single-centre tests with one kind of entity. The sagitta operates as a single continuous mechanism, from the microscopic singular Green's function generator to the relational pairwise ledger; this relational arrival geometry defines three regions governed by a single continuous equation of geometry and energy, with opposed pairs of streams switching on at $\sqrt{2}$ body radii to soften dynamic terms on bodies entering Region 2. At first order the sagitta map and the rod-clock readout are joined by an exact dictionary that the source distribution generates itself. The dynamics are two non-overlapping operations, projection and relaxation; relaxation follows from additivity and universal confinement, giving the exponential response and $\beta = 1$; $\gamma = 1$, frame dragging, the absence of aberration for uniformly moving sources, tensor radiation, and a single local light cone for light and ripples follow from the kernel. Mercury's $43.0''/\text{century}$, light deflection, the Shapiro delay, the geoid and geodetic precession follow from the same readout, and the interior joins the exterior without a matching step.

Across all tested regimes, the framework is empirically equivalent to General Relativity, and its claims there are of explanatory economy; the multi-body sector closes via pairwise triad chains ($N(N-1)$ directed channels), guaranteeing exact bilateral action-reaction and $\xi = \eta = 0$. Its distinctive content lies in its ontology, in the criterion that only confined flux sources debt, in the derivation of the exterior by direct integration, and in the conservation axiom combined with the absence of self-sourcing. Physical departures from General Relativity are strictly demarcated to near-limb Region 2 dynamic softening and asymptotic strong-field compression, where clocks slow and matter compresses without ever encountering an event horizon or geodesic singularity.

A Momentum Flux Mathematics

This appendix collects the results that follow from geometry and conservation alone. Throughout, $L = |\mathbf{x} - \mathbf{x}'|$ and $\hat{\mathbf{n}} = (\mathbf{x} - \mathbf{x}')/L$.

A.1 Shell Averages and the Monopole

Lemma A.1 (Shell averages). *For a thin spherical shell of radius a and a field point at distance r from its centre, the shell averages of $1/L$ and L are*

$$\left\langle \frac{1}{L} \right\rangle = \frac{1}{\max(r, a)}, \quad \langle L \rangle = \begin{cases} r + \frac{a^2}{3r}, & r \geq a, \\ a + \frac{r^2}{3a}, & r < a. \end{cases} \quad (78)$$

Proof. With μ the cosine of the angle at the centre, $L^2 = r^2 + a^2 - 2ra\mu$, so $d\mu = -L dL/(ra)$ and the average over $\mu \in [-1, 1]$ of a function $f(L)$ is $\frac{1}{2ra} \int_{|r-a|}^{r+a} f(L) L dL$. For $f = 1/L$ this is $\frac{2 \min(r, a)}{2ra} = 1/\max(r, a)$. For $f = L$ it is $[(r+a)^3 - |r-a|^3]/(6ra)$, which gives the two cases. $\square$

Theorem A.2 (Monopole by direct integration). *For a spherically symmetric source of total confined energy $E = Mc^2$ and physical radius R , the summed static debt at $r \geq R$ is exactly $\epsilon = GM/c^2 r$, and the drift $c^2 \nabla \epsilon$ is exactly $-GM \hat{\mathbf{r}}/r^2$, with no multipole error.*

Proof. By Eq. (11), $\epsilon = (G/c^4) \int dE/L$. Decompose the source into shells; by Lemma A.1 each shell of energy dE and radius $a \leq R \leq r$ contributes $(G/c^4) dE/r$. Summing gives $GE/c^4 r = GM/c^2 r$. $\square$

Remark A.3. The same result follows from Gauss's theorem applied to the net flux $\mathbf{N}$, which is divergence-free outside the source. The direct integral is preferred here because it is the integral of the Green's function itself, and because the same shell averages give the arrival tensor, which Gauss's theorem cannot reach.

A.2 The Arrival Tensor

Theorem A.4 (Arrival tensor outside a spherical source). *For a spherically symmetric source with energy-weighted mean square radius $\langle r'^2 \rangle = \frac{1}{E} \int r'^2 dE$, the arrival tensor at $r \geq R$ is exactly Eq. (26).*

Proof. By the bridge identity (Eq. 28), $T_{ij} = \epsilon \delta_{ij} - \partial_i \partial_j \Xi$ with $\Xi = (G/c^4) \int dE L$. By Lemma A.1, outside the source $\Xi = (GE/c^4)(r + \langle r'^2 \rangle/3r)$. Using $\partial_i \partial_j r = (\delta_{ij} - \hat{r}_i \hat{r}_j)/r$ and $\partial_i \partial_j (1/r) = (3\hat{r}_i \hat{r}_j - \delta_{ij})/r^3$,

$$T_{ij} = \frac{GE}{c^4} \left[\frac{\delta_{ij}}{r} - \frac{\delta_{ij} - \hat{r}_i \hat{r}_j}{r} - \frac{\langle r'^2 \rangle}{3} \frac{3\hat{r}_i \hat{r}_j - \delta_{ij}}{r^3} \right], \quad (79)$$

which is Eq. (26). Higher multipoles vanish because $\nabla^2 L = 2/L$ is harmonic outside the source, so the shell average of L terminates. $\square$

For the Sun, standard solar models [55] give a normalized moment of inertia $I/(M_\odot R_\odot^2) \approx 0.059$, so $\langle r'^2 \rangle \approx 0.0885 R_\odot^2$, and the arrival-tensor anisotropy peaks inside the Sun.

A.3 The Tensor Conservation Law

For one point source, the momentum-flux tensor of its stream, $F_{ij} = \hat{n}_i \hat{n}_j / L^2 = x_i x_j / L^4$ (with $\mathbf{x}$ measured from the source), is divergence-free away from the source:

$$\partial_j F_{ij} = \frac{x_i}{L^4} + \frac{3x_i}{L^4} - \frac{4x_i}{L^4} = 0. \quad (80)$$

The sum over any distribution is therefore divergence-free outside the sources. For a spherically symmetric sum, $F_{ij} = A \hat{r}_i \hat{r}_j + B (\delta_{ij} - \hat{r}_i \hat{r}_j)$, and the radial component of $\partial_j F_{ij} = 0$ is Eq. (27).

A.4 The Absolute-Net Identity and Closure at Directed Pairs

With $I = \sum_b w_b$ and $\mathbf{N} = \sum_b w_b \hat{\mathbf{n}}_b$, $I^2 = \sum_{b,c} w_b w_c$ and $|\mathbf{N}|^2 = \sum_{b,c} w_b w_c \cos \theta_{bc}$, so Eq. (23) holds identically across the $N(N-1)$ directed pairs (with diagonal $b = c$ vanishing). A finite set of unit vectors in $\mathbb{R}^3$ is determined up to an orthogonal transformation by its Gram matrix $\cos \theta_{bc}$; any rotation-invariant function of the arrival directions is therefore a function of the pairwise angles, up to the sign of triple products, which reverses under reflection.

A.5 Onset of Opposed Pairs

From a field point at distance r outside a sphere of radius R , every arrival direction lies within the cone of half-angle $\arcsin(R/r)$ about the radial line, bounded by the tangent lines to the limb. The widest angle between two arrival directions is therefore $\Theta = 2 \arcsin(R/r)$, which exceeds 90° if and only if $R/r > 1/\sqrt{2}$, that is, $r < \sqrt{2}R$. For a thin shell of radius a the boundary is $\sqrt{2}a$, so a body's Region 2 is set by its outer layers. In a fully isotropic bath, the share of $I^2 - |\mathbf{N}|^2$ supplied by opposed pairs is

$$\frac{\int_{-1}^0 (1-c) dc}{\int_{-1}^1 (1-c) dc} = \frac{3/2}{2} = \frac{3}{4}. \quad (81)$$

A.6 The Bridge Identity and the Synchronization Gradient

For one stream, $\partial_i L = \hat{n}_i$ and $\partial_i \partial_j L = (\delta_{ij} - \hat{n}_i \hat{n}_j)/L$, so $\hat{n}_i \hat{n}_j/L = \delta_{ij}/L - \partial_i \partial_j L$. Integrating against $(G/c^4) dE$ gives Eq. (28). Likewise $\partial_i \ln L = \hat{n}_i/L$, which gives Eq. (17), since for a static stream $h_{0i} = 2\epsilon_j \ell_0 \ell_i = -2\epsilon_j \hat{n}_i$.

A.7 The Retarded Distance of a Uniformly Moving Source

Let a source move as $\mathbf{x}_s(t') = \mathbf{x}_0 + \mathbf{v}t'$, with $c = 1$, and let t' be the retarded time for the field point $(t, \mathbf{x})$, so that $|\mathbf{x} - \mathbf{x}_s(t')| = t - t'$. With $R = (t - t', \mathbf{x} - \mathbf{x}_s(t'))$ and $u = \gamma(1, \mathbf{v})$,

$$\rho = -u \cdot R = \gamma[(t - t') - \mathbf{v} \cdot (\mathbf{x} - \mathbf{x}_s(t'))]. \quad (82)$$

Writing $\mathbf{x} - \mathbf{x}_s(t') = \mathbf{d} + \mathbf{v}(t - t')$, with $\mathbf{d} = \mathbf{x} - \mathbf{x}_s(t)$ the separation from the present position, and eliminating $t - t'$ with the retarded condition gives $\rho = \gamma\sqrt{d^2 - |\mathbf{v} \times \mathbf{d}|^2}$, the Liénard-Wiechert result restated as Eq. (16). I checked it numerically at random field points against the retarded-time solution, with agreement to ten digits.

A.8 The Virial Identity

With $c = 1$, local conservation gives $\partial_t T^{00} + \partial_k T^{0k} = 0$ and $\partial_t T^{0i} + \partial_k T^{ik} = 0$. For a structure bounded within a region whose surface carries no flux,

$$\begin{aligned} \frac{d^2}{dt^2} \int T^{00} x^i x^j dV &= - \int \partial_t \partial_k T^{0k} x^i x^j dV = \int \partial_t (T^{0i} x^j + T^{0j} x^i) dV \\ &= - \int (\partial_k T^{ik} x^j + \partial_k T^{jk} x^i) dV = 2 \int T^{ij} dV, \end{aligned} \quad (83)$$

which is Eq. (44).

A.9 The Relative-Debt Functional Equation

Relative debt requires $f(\epsilon_{bg} + \delta\epsilon) = f(\epsilon_{bg}) f(\delta\epsilon)$ for all ϵ_{bg} , with $f(0) = 1$. The only continuous solutions are $f(\epsilon) = e^{k\epsilon}$. The first-order cadence, $f \approx 1 - 2\epsilon$, fixes $k = -2$, giving Eq. (31).

A.10 The Eikonal Light Cone

Theorem A.5 (Eikonal light cone). *At first post-Newtonian order in the areal chart, high-frequency waves satisfying $\tilde{g}^{\mu\nu}\partial_\mu S\partial_\nu S = 0$ move radially at $v_r = c\sqrt{(1-2\epsilon)(1-2\zeta)}$ (reducing to $c(1-2\epsilon)$ in the exterior) and transversely at $v_\perp = c\sqrt{1-2\epsilon}$, the null directions of Eq. (57).*

Proof. With $\tilde{g}^{\mu\nu} = \text{diag}(-(1-2\epsilon)^{-1}, 1-2\zeta, r^{-2}, (r\sin\theta)^{-2})$,

$$-\frac{1}{1-2\epsilon}\frac{1}{c^2}(\partial_t S)^2 + (1-2\zeta)(\partial_r S)^2 + \frac{1}{r^2}(\partial_\theta S)^2 + \frac{1}{r^2\sin^2\theta}(\partial_\phi S)^2 = 0. \quad (84)$$

For radial rays, $|\partial_t S/\partial_r S|^2 = c^2(1-2\epsilon)(1-2\zeta)$; for transverse rays, $|r\,d\phi/dt|^2 = c^2(1-2\epsilon)$. $\square$

A.11 Parity and Conservation for a Spherically Symmetric Source

For any spherically symmetric source distribution undergoing arbitrary radial pulsations, the exterior momentum flux is constrained by local conservation $\partial_\mu \Pi^{\mu\nu} = 0$ and spherical parity.

Decomposing the retarded potential of the momentum flux into spherical multipoles, the mass current $T^{0i} = \rho c v^i(\mathbf{x}', t)$ is purely radial ($v^i \propto \hat{r}^i$). The dipole moment of the source vanishes identically because total linear momentum is conserved ($\mathbf{P} = \int T^{0i} d^3x' = \mathbf{0}$ in the centre-of-mass frame). For a spherically symmetric pulsation, all higher multipoles ($\ell \geq 1$) vanish identically by spherical integration parity:

$$\int_{S^2} \hat{r}^i d\Omega = 0, \quad \int_{S^2} (\hat{r}^i \hat{r}^j - \frac{1}{3}\delta^{ij}) d\Omega = 0. \quad (85)$$

Furthermore, because total momentum flux is locally conserved and no net mass-energy leaves the boundary in a radial pulsation, the monopole moment $\int T^{00} d^3x' = Mc^2$ is strictly constant in time. Consequently, time-dependent monopole radiation is forbidden by construction, and the exterior scalar debt remains strictly static: $\epsilon(r) = GM/(c^2 r)$.

A.12 Numerical Checks

Table 13: Numerical Checks Performed for This Version.

Check	Method	Agreement
Arrival tensor, Eq. (26)	Direct sum over 6×10^6 points, uniform and concentrated profiles, $r = 1.2\text{--}4R$	4–5 digits
Tensor conservation, Eq. (27)	Shell quadrature, uniform sphere, $r = 1.1\text{--}3R$	$10^{-8}\text{--}10^{-10}$
Opposed-pair onset at $\sqrt{2}R$	Pairwise sum over 5000 points	Confirms analytic $\sqrt{2}R$ threshold within sampling resolution ($\Delta r/R \approx 0.02$)
Bridge identity, Eq. (28)	Lopsided source, finite-difference Hessian	2×10^{-7}
Retarded distance, Eq. (16)	Random field points vs. retarded-time root	10 digits
Areal compensation, Eq. (52)	Two profiles, interior and exterior	4 decimals (Table 11)
Fold-back error	Self-consistent iteration, uniform sphere	Scales as $(GM/c^2 R)^2$

B Summary Tables

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Table 14: Epistemic Classification of the Framework.

Result / Sector	Origin in Framework	Status
<i>I. Results (through first post-Newtonian order)</i>		
Mercury perihelion advance	Relaxation of an orbiting assembly; unshifted $p_\phi = mL$	43.0''/century ($\mathcal{C} = 2_{\text{time}} + 1_{\text{space}} = 3$)
Light deflection	Fermat's principle across the light cone	1.75'' to $\mathcal{O}(\epsilon)$
Shapiro delay	Path integration across the light cone	Matches Cassini
Geoid cadence	Moving-loop proper time on a hydrostatic surface	Uniform to first order
Geodetic precession	Transport of a circulation axis	Matches Gravity Probe B
Exterior monopole	Direct integration of the Green's function	Exact for any spherical profile
Arrival tensor	Direct integration; one terminating term	Exact; checked numerically
Bridge between branches	$T_{ij} = \epsilon \delta_{ij} - \partial_i \partial_j \Xi$	Exact for any distribution
No aberration (uniform motion)	Retarded distance depends on present position	Exact
No preferred frame	Covariant kernel	$\alpha_1 = \alpha_2 = 0$ at linear order
One light cone	All unconfined flux at the local speed (Axiom III)	Consistent with GW170817
Relational departure from Birkhoff: arrival tensor $\langle r'^2 \rangle$	Direct integration of T_{ij}	Breaks Birkhoff; preserves interior memory
Frame dragging	Momentum part of the kernel	Matches Gravity Probe B
Wave polarizations	Conserved source	Two tensor modes
Universality of free fall	Universal confinement response with persistence and conservation; stationary phase	Follows from the response rule
$C = 2$	Kinematic derivative of quadratic speed budget	Derived
$\gamma = 1$	Null dyad of momentum flux and rod-clock duality	Derived
$\beta = 1$	Universal confinement response	Derived
Interior spatial metric	Areal compensation	$g_{rr} = 1 + 2\zeta$ at first order
<i>II. Axioms, Rules and Calibrations</i>		
Axioms I-III	Flat substrate; confined flux; additivity, conservation, relative debt, unify then read out	Postulated
Projection rule	Green's function; $1/L$ and $\ell \otimes \ell$ fixed by additivity and c	Largely derived; source criterion and G as input
Response rule	Universal confinement with relative debt	Gives the exponential
Calibration	G	As in General Relativity
<i>III. Empirical Demarcation and Frontiers</i>		
Near-limb Region 2 dynamic softening	Opposing limb curl cancellation ($\Theta > 90^\circ$)	Softens bulk acceleration; deposits stress/dilation
Asymptotic strong-field compression	Exponential response $c_{\text{circ}} = c e^{-\epsilon}$	No horizon or singularity; continuous slowing
$N(N-1)$ pairwise simulation	Divergence-free flux ledger	Running-code verification
EHT and ringdown confrontation	Horizonless compression phenomenology	Observational test against horizon models
Magnitude of G	Structure of matter	Open

Table 15: Structural Comparison between General Relativity and the Momentum-Flux Framework.

Characteristic	General Relativity	Momentum-Flux Framework (This Work)
Kinds of entity	Matter and geometry	Momentum flux only
Spacetime foundation	Curved manifold $(\mathcal{M}, g_{\mu\nu})$	Flat Minkowski $(\mathbb{R}^{1,3}, \eta_{\mu\nu})$
Exterior of a sphere	Birkhoff's theorem	Static monopole by direct integration; arrival geometry breaks Birkhoff via $\langle r'^2 \rangle$
Stellar description	Schwarzschild and TOV, matched at the surface	One integral; continuity automatic
Vacuum field	Non-linear self-gravitation	Field sources no debt; debts add on the substrate
Second-order structure	Nonlinear field equations	Relaxation applied causally; singular generator and relational pairwise channels
Coordinate differences	Gauge	Physical substrate-readout displacement, unseen at first order
Preferred frame	None	None at linear order
Wave polarizations	Two tensor	Two tensor
Conservation	$\nabla_\mu T^{\mu\nu} = 0$; totals in special cases	Local and exact global conservation on the flat substrate
Nature of attraction	Geodesic motion in curved spacetime	Stationary phase: drift toward slower clocks
Symmetric curl	Enters through energy and stress	Explicit sagitta residual: singular Green's function and relational pairwise ledger
Clock cadence	Metric component g_{00}	Transit time of circulating flux
Precession attribution	Isotropic: $1_{\text{time}} + 2_{\text{space}}$	Areal: $2_{\text{time}} + 1_{\text{space}}$
Exterior beyond 1PN	Schwarzschild	Exponential: asymptotic compression, no horizon (physical departure)
Singular collapse	Geodesic incompleteness	No geometric singularity
Interior spatial metric	$(1 - 2Gm/c^2 r)^{-1}$	$1 + 2\zeta$ at first order; $(1 - \zeta)^{-2}$ under the exponential

Table 16: Principal Operators of Momentum Flux Mathematics.

Operator	Definition	Interpretation
Singular sagitta	$h = s^2/2L$, curvature $\kappa = 1/L$	One circulation's residual at a field point (Green's function generator)
Green's kernel $\mathcal{K}_{\mu\nu}$	$\frac{2GE^{\text{rel}}}{c^4} \frac{\ell_\mu \ell_\nu}{\rho}$	Projection plus coupling half of reception
Static debt	$\epsilon = (G/c^4) \int dE/L$	Unified scalar debt
Arrival tensor	$T_{ij} = (G/c^4) \int dE \hat{n}_i \hat{n}_j / L$	Direction-resolved debt
Bridge	$T_{ij} = \epsilon \delta_{ij} - \partial_i \partial_j \Xi$, $\Xi = (G/c^4) \int dE L$	Dictionary between the branches
Synchronization	$h_{0i} = -\partial_i \Lambda$, $\Lambda = (2G/c^4) \int dE \ln L$	Removed by resynchronization
Relational sagitta	$I^2 - \mathbf{N} ^2 = \sum w_b w_c (1 - \cos \theta_{bc})$	Summed pairwise arrival geometry; transverse hoop stress
Momentum-flux tensor	$A' + 2(A - B)/r = 0$	Radial flux traded for hoop stress
Limb opposition	$\chi = 1 - 2R^2/r^2$	Opposed pairs where $\chi < 0$
Retarded distance	$\rho = \gamma \sqrt{d^2 - \mathbf{v} \times \mathbf{d} ^2}$	No aberration for uniform motion
Response	$(d\tau/dt)^2 = e^{-2\epsilon}$, $dl_{\text{phys}} = e^\epsilon dl$	Universal confinement
Speed budget	$c^2(1 - 2\epsilon) = c_{\text{circ}}^2 + v^2$ (first order)	Cadence, translation and debt
Areal compensation	$g_{rr} - 1 = 2[T_{rr} - T_{\perp} - rT'_{\perp}]$	Rods reading the arrival tensor
Local streaming	$\zeta = Gm/c^2 \bar{r}$	Enclosed radial streaming
Virial identity	$\int T^{ij} dV = \frac{1}{2} \partial_t^2 \int T^{00} x^i x^j dV$	Persistence gives weight = inertia
Local conservation	$\partial_\mu \Pi_{\text{total}}^{\mu\nu} = 0$	Matter plus propagating flux

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